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Spacecraft maneuvering via atmospheric differential drag using an adaptive Lyapunov controller

(AAS 13-440)

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Outline

- Introduction
- Linear reference model and Nonlinear Model
- Lyapunov Approach
- Drag devices activation strategy
- Critical value for the magnitude of differential drag
- Adaptive Lyapunov Control strategy
- Numerical Simulations: Re-Phase, Fly-Around, and Rendezvous

See also:

Perez, D., Bevilacqua, R., "Differential Drag Spacecraft Rendezvous using an Adaptive Lyapunov Control Strategy", Acta Astronautica 83 (2013) 196–207 http://dx.doi.org/10.1016/j.actaastro.2012.09.005.

(winner of best student paper award at 1st International Academy of Astronautics Conference on Dynamics and Control of Space Systems)

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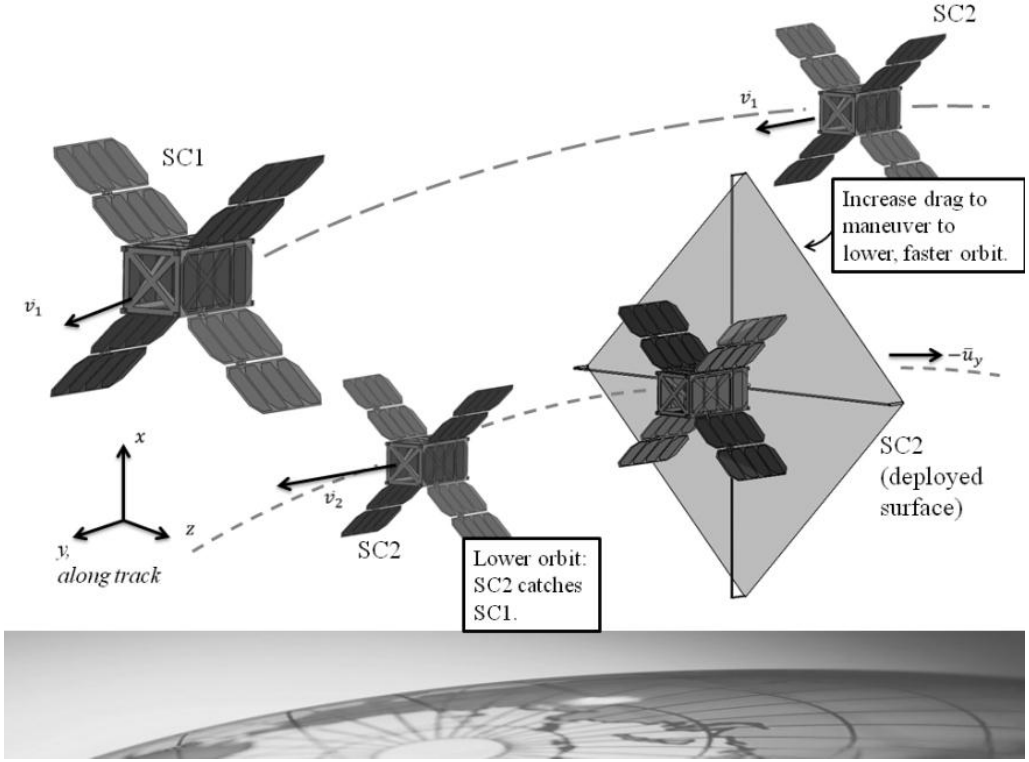
- Differential in the aerodynamic drag produces a differential in acceleration
- This differential can be used to control the relative motion of the S/C on the orbital plane only
- It is assumed that that the drag devices act instantly (on-off control)
- Control systems for drag maneuvers must cope with many uncertainties (density changes, winds, contact dynamics, etc.).



Linear Reference Model

- The Schweighart and Sedwick model is used to create the stable reference model
- LQR controller is used to stabilize the Schweighart and Sedwick model
- The resulting reference model is described by:

$$\dot{\boldsymbol{x}}_d = \underline{\boldsymbol{A}}_d \boldsymbol{x}_d + \underline{\boldsymbol{B}}\boldsymbol{u}_d, \quad \underline{\boldsymbol{A}}_d = \underline{\boldsymbol{A}} - \underline{\boldsymbol{B}}\underline{\boldsymbol{K}}, \quad \boldsymbol{x}_d = \begin{bmatrix} x_d & y_d & \dot{x}_d & \dot{y}_d \end{bmatrix}^T,$$

$$\boldsymbol{u}_d = \underline{\boldsymbol{K}}\boldsymbol{x}_t$$

- $\underline{\boldsymbol{K}}$ is found by solving the LQR problem

- The dynamics of S/C relative motion are nonlinear due to
  - J_2 perturbation
  - Variations on the atmospheric density at LEO
  - Solar pressure radiation
  - Etc.
- The general expression for the real world nonlinear dynamics, including nonlinearities is:

$$\dot{\mathbf{x}} = \boldsymbol{f}(\mathbf{x}) + \underline{\mathbf{B}}\mathbf{u}, \quad \mathbf{x} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T, \qquad u = \begin{cases} a_{Drel} \\ 0 \\ -a_{Drel} \end{cases}$$

- A Lyapunov function of the tracking error is defined as: $V = \mathbf{e}^T \underline{\mathbf{P}}\mathbf{e}, \qquad \mathbf{e} = \mathbf{x} - \mathbf{x}_d, \qquad \underline{\mathbf{P}} \succ 0$
- After some algebraic manipulation, the time derivative of the Lyapunov function is:

$$\dot{V} = \mathbf{e}^T (\underline{\mathbf{A}}_d{}^T \underline{\mathbf{P}} + \underline{\mathbf{P}}\underline{\mathbf{A}}_d)\mathbf{e} + 2\mathbf{e}^T \underline{\mathbf{P}}(\mathbf{f}(\mathbf{x}) - \underline{\mathbf{A}}_d \mathbf{x} + \underline{\mathbf{B}} a_{Drel} \hat{u} - \underline{\mathbf{B}}\mathbf{u}_d)$$

- Defining $\underline{\mathbf{A}}_d$ Hurwitz and $\underline{\mathbf{Q}}$ symmetric positive definite, $\underline{\mathbf{P}}$ can be found using:

$$-\underline{\mathbf{Q}} = \underline{\mathbf{A}}_d{}^T \underline{\mathbf{P}} + \underline{\mathbf{P}}\underline{\mathbf{A}}_d$$

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- Rearranging $\dot{V}$ yields

$$\dot{V} = -\boldsymbol{e}^T \underline{\boldsymbol{Q}}\boldsymbol{e} + 2(\beta\hat{u} - \delta), \quad \hat{u} = \begin{cases} 1 \\ 0 \\ -1 \end{cases},$$

$$\beta = \boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}} a_{Drel}, \quad \delta = -\boldsymbol{e}^T \underline{\boldsymbol{P}} \left(\underline{\boldsymbol{A}}_d \boldsymbol{x} - \boldsymbol{f}\left(\boldsymbol{x}\right) + \underline{\boldsymbol{B}}\boldsymbol{u}_d \right)$$

- Guaranteeing $\dot{V} < 0$ would imply that the tracking error ($\boldsymbol{e}$) converges to zero
- By selecting:

$$\hat{u} = -sign(\beta) = -sign(\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}})$$

$\dot{V}$ is ensured to be as small as possible.

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Critical value for the magnitude of differential drag acceleration

- Product $\beta\hat{u}$ is the only controllable term that influences the behavior of $\dot{V} = 2\left(\beta\hat{u} - \delta\right)$
- There must be a minimum value for a_{Drel} that allows for $\dot{V}$ to be negative for given values of β and δ
- This value is found analytically by solving:

$$0 \geq \boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}} a_{Drel} \hat{u} - \delta$$

- Solving this expression for a_{Drel} yields

$$a_{Drel} \geq \frac{\delta}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|} = \frac{\boldsymbol{e}^T \underline{\boldsymbol{P}}\left(\underline{\boldsymbol{A}}_d \boldsymbol{x} - \boldsymbol{f}\left(\boldsymbol{x}\right) + \underline{\boldsymbol{B}}\boldsymbol{u}_d\right)}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|} = a_{Dcrit}$$

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- Choosing appropriate values for the entries of $\underline{\boldsymbol{Q}}$ and $\underline{\boldsymbol{A}}_d$ can reduce a_{Dcrit}
- To achieve this, the following partial derivatives were developed

$$\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{A}}_d}, \quad \frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{Q}}}$$

- Starting from the general case of the critical value

$$a_{Dcrit} = \frac{\boldsymbol{e}^T \underline{\boldsymbol{P}}\left(\underline{\boldsymbol{A}}_d \boldsymbol{x} - \boldsymbol{f}\left(\boldsymbol{x}\right) + \underline{\boldsymbol{B}}\boldsymbol{u}_d\right)}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|}$$

- The Lyapunov equation was transformed into:

$$-\underline{\boldsymbol{Q}} = \underline{\boldsymbol{A}}_d{}^T \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_d, \quad \underline{\boldsymbol{A}}_v \boldsymbol{P}_v = -\boldsymbol{Q}_v,$$

$$\underline{\boldsymbol{A}}_v = \underline{\mathbf{I}}_{4x4} \otimes \underline{\boldsymbol{A}}_d + \underline{\boldsymbol{A}}_d \otimes \underline{\mathbf{I}}_{4x4}, \quad \boldsymbol{P}_v = vec(\underline{\boldsymbol{P}}), \quad \boldsymbol{Q}_v = vec(\underline{\boldsymbol{Q}}),$$

$$\boldsymbol{P}_v = -\underline{\boldsymbol{A}}_v{}^{-1} \boldsymbol{Q}_v$$

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- Using $\boldsymbol{P}_v = -\underline{\boldsymbol{A}}_v^{-1}\boldsymbol{Q}_v$

The following derivatives were found in previous work

$$\frac{\partial \underline{\boldsymbol{P}}}{\partial \underline{\boldsymbol{A}}_d} = \mathrm{T}_2\left(\left[\mathrm{T}_3^{-1}\left(\frac{\partial \underline{\boldsymbol{A}}_v}{\partial \underline{\boldsymbol{A}}_d}\right)\otimes \underline{\mathbf{I}}_{16x16}\right]\left[\underline{\mathbf{I}}_{4x4}\otimes \mathrm{T}_1^{-1}\left(\frac{\partial \boldsymbol{P}_v}{\partial \underline{\boldsymbol{A}}_v}\right)\right]\right),$$

$$\frac{\partial \underline{\boldsymbol{A}}_v}{\partial \underline{\boldsymbol{A}}_d} = \left(\underline{\mathbf{I}}_{4x4}\otimes \underline{\boldsymbol{U}}_1\right)\left(\underline{\boldsymbol{U}}_{4x4}\otimes \underline{\mathbf{I}}_{4x4}\right)\left(\underline{\mathbf{I}}_{4x4}\otimes \underline{\boldsymbol{U}}_1\right)+\underline{\boldsymbol{U}}_{4x4}\otimes \underline{\mathbf{I}}_{4x4},$$

$$\frac{\partial \boldsymbol{P}_v}{\partial \underline{\boldsymbol{A}}_v} = \left(\underline{\mathbf{I}}_{16x16}\otimes \underline{\boldsymbol{A}}_v^{-1}\right)\underline{\boldsymbol{U}}_{16x16}\left(\underline{\mathbf{I}}_{16x16}\otimes \underline{\boldsymbol{A}}_v^{-1}\right)\left(\underline{\mathbf{I}}_{16x16}\otimes \boldsymbol{Q}_v\right), \quad \frac{\partial \underline{\boldsymbol{P}}}{\partial \underline{\boldsymbol{Q}}} = \mathrm{T}_1\left(\left(-\underline{\boldsymbol{A}}_v^{-1}\right)^T\right)$$

- Using the chain rule the desired final expressions can be found:

$$\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{Q}}} = \mathrm{T}_3^{-1}\left(\mathrm{T}_1\left(\left(-\underline{\boldsymbol{A}}_v^{-1}\right)^T\right)\right)\left[\underline{\mathbf{I}}_{4x4}\otimes \mathrm{T}_1^{-1}\left(\frac{\partial \eta(\underline{\boldsymbol{P}})}{\partial \underline{\boldsymbol{P}}}\right)\right]\left[\underline{\mathbf{I}}_{4x4}\otimes\left(\underline{\boldsymbol{A}}_d\boldsymbol{x}-\boldsymbol{f}(\boldsymbol{x})+\underline{\boldsymbol{B}}\boldsymbol{u}_d\right)\right],$$

$$\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{A}}_d} = \mathrm{T}_3^{-1}\left(\frac{\partial \underline{\boldsymbol{P}}}{\partial \underline{\boldsymbol{A}}_d}\right)\left[\underline{\mathbf{I}}_{4x4}\otimes \mathrm{T}_1^{-1}\left(\frac{\partial \eta(\underline{\boldsymbol{P}})}{\partial \underline{\boldsymbol{P}}}\right)\right]\left[\underline{\mathbf{I}}_{4x4}\otimes\left(\underline{\boldsymbol{A}}_d\boldsymbol{x}-\boldsymbol{f}(\boldsymbol{x})+\underline{\boldsymbol{B}}\boldsymbol{u}_d\right)\right]+$$
$$\left[\underline{\mathbf{I}}_{4x4}\otimes\frac{\boldsymbol{e}^T\underline{\boldsymbol{P}}}{\left|\boldsymbol{e}^T\underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|}\right]\underline{\boldsymbol{U}}_{16x16}(\underline{\mathbf{I}}_{4x4}\otimes\boldsymbol{x}),$$

$$\frac{\partial \eta(\underline{\boldsymbol{P}})}{\partial \underline{\boldsymbol{P}}} = (\underline{\mathbf{I}}_{4x4}\otimes\boldsymbol{e}^T)\underline{\boldsymbol{U}}_{16x16}\left(\underline{\mathbf{I}}_{4x4}\otimes\frac{\underline{\mathbf{I}}_{4x4}}{\left|\boldsymbol{e}^T\underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|}\right)-(\underline{\mathbf{I}}_{4x4}\otimes\boldsymbol{e}^T\underline{\boldsymbol{P}})\left[\frac{\left(\boldsymbol{e}^T\underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right)\left(\boldsymbol{e}^T\underline{\boldsymbol{B}}\right)}{\left|\boldsymbol{e}^T\underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|^3}\right]$$

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- Using these derivatives $\underline{\boldsymbol{A}}_{\boldsymbol{d}}$ and $\underline{\boldsymbol{Q}}$ are adapted as follows:

$$\frac{dA_{ij}}{dt} = \kappa_A \left[-sign\left(\frac{\partial a_{Dcrit}}{\partial A_{ij}}\right)\delta_A \right], \quad \frac{dQ_{ij}}{dt} = \kappa_Q \left[-sign\left(\frac{\partial a_{Dcrit}}{\partial Q_{ij}}\right)\delta_Q \right]$$

$$\kappa_A = \begin{cases} 1 \text{ if } \dfrac{\partial a_{Dcrit}}{\partial A_{ij}} > \dfrac{\partial a_{Dcrit}}{\partial A_{kl}} \text{ for } i, j \neq k, l \\ 0 \text{ else} \end{cases}, \quad \kappa_Q = \begin{cases} 1 \text{ if } \dfrac{\partial a_{Dcrit}}{\partial Q_{ij}} > \dfrac{\partial a_{Dcrit}}{\partial Q_{kl}} \text{ for } i, j \neq k, l \\ 0 \text{ else} \end{cases}$$

- These were designed such that:
  - $\underline{\boldsymbol{Q}}$ is symmetric positive definite
  - $\underline{\boldsymbol{A}}_{\boldsymbol{d}}$ is Hurwitz
- These adaptations result in an adaptation of the quadratic Lyapunov function

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- Simulations in STK using High-Precision Orbit Propagator (HPOP)
  - Full gravitational field model
  - Variable atmospheric density (using NRLMSISE-00)
  - Solar pressure radiation effects

Parameter	Value
Second zonal harmonic J_2	1.08E-03
Radius of the Earth R (km)	6378.1363
Gravitational parameter μ (km^3/sec^2)	398600.4418
Target's inclination (deg)	98
Target's semi-major axis (km)	6778
Target's right ascension of the ascending node (deg)	262
Target's argument of perigee (deg)	30
Target's true anomaly (deg)	25
Target's eccentricity	0
v_s (km/sec)	7.68
m(kg)	10
S_{min} surface withheld (m^2)	0.5
S_{max} surface deployed (m^2)	2.5
C_{Dmin}	1.5
C_{D0}	2
C_{Dmax}	2.5

- The maneuver ended when S/C were within 10m.

Parameter	Rendezvous	Fly-Around	Re-Phase
x (km)	-1	0	0
y (km)	-2	-4.25	-1.9
$\dot{x}$ (km/sec)	4.8E-007	0	0
$\dot{y}$ (km/sec)	1.70E-04	0	0

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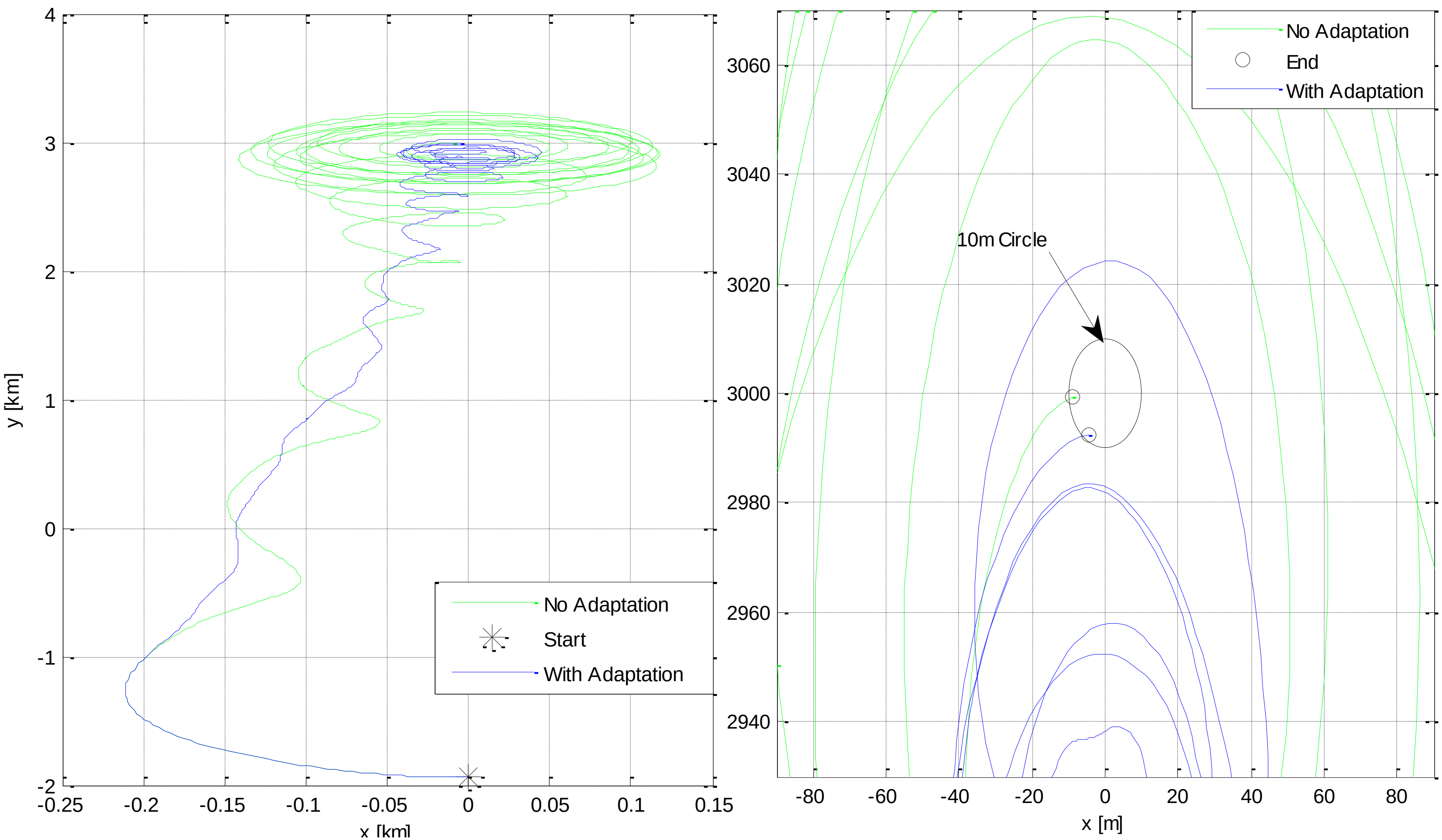
Numerical Simulations: Re-Phase

- Simulated trajectory in the x-y plane



- Initial relative position of 0 km in x, -1.9 km in y in the LVLH
- Final relative position of 0 km in x, 3 km in y in the LVLH

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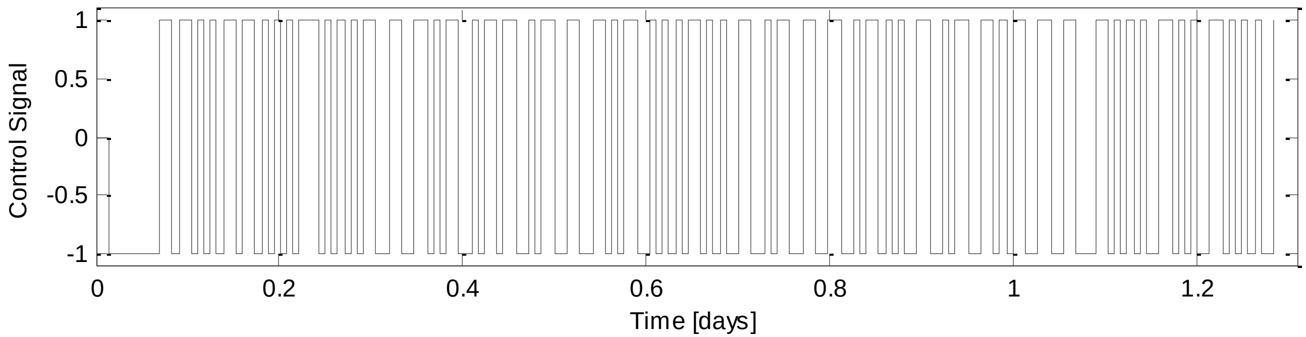
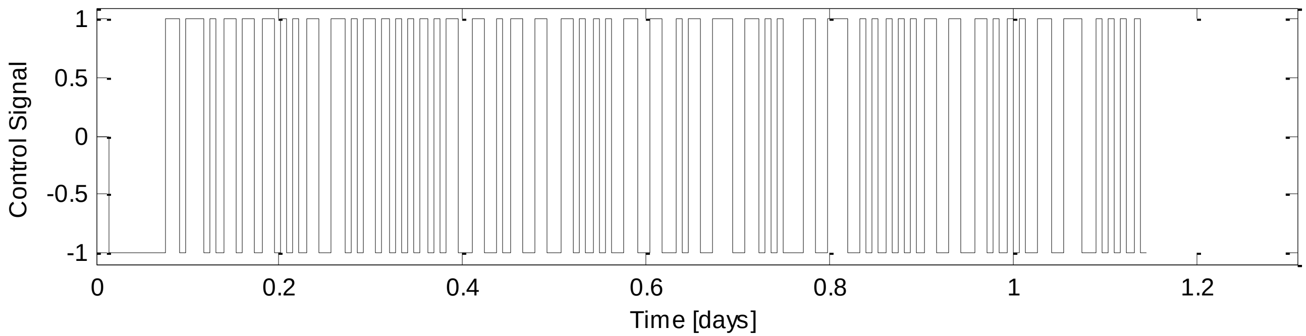
Numerical Simulations: Re-Phase

- Control signal for both controllers



No Adaptation



With Adaptation

- Adaptive VS Non Adaptive
  - Number of control switches: 107 VS 124 (13.7%, less actuation)
  - Maneuver time: 27 hr VS 31 hr (10.9%, less time)

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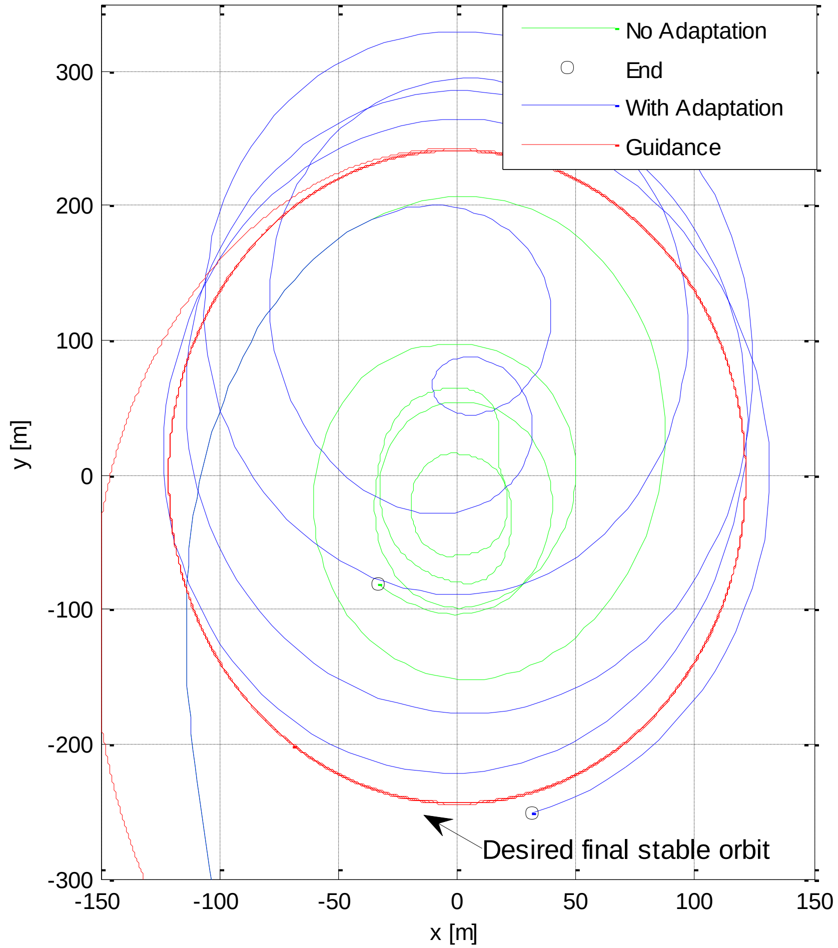
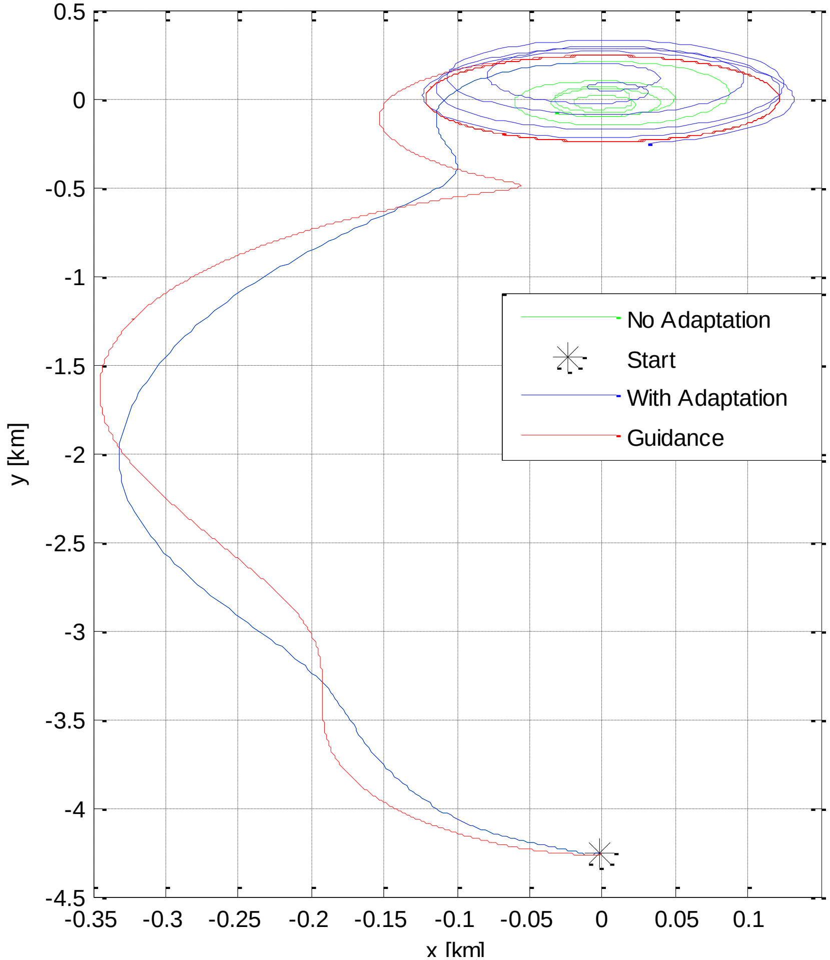
Numerical Simulations: Fly-Around

- Simulated trajectory in the x-y plane

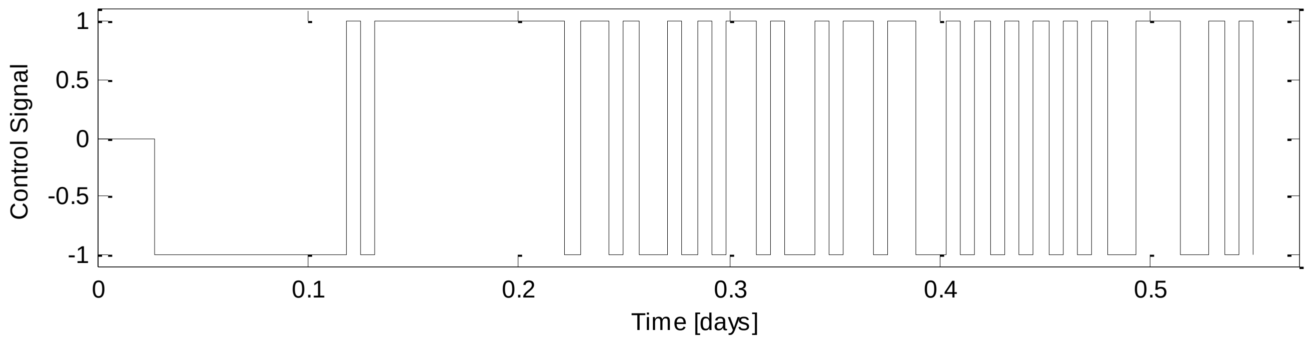




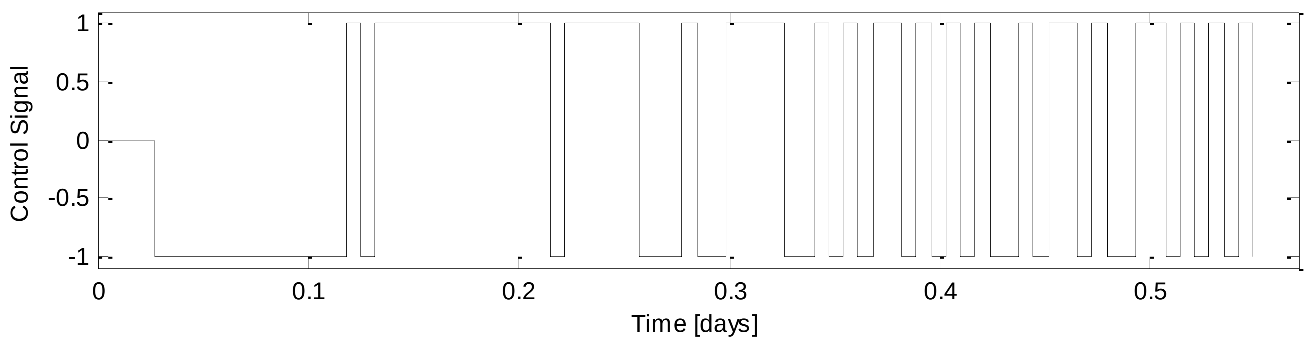
- Initial relative position of 0 km in x, -4.25 km in y in the LVLH
- Final State: Stable orbit around Target S/C

Numerical Simulations: Fly-Around

- Control signal for both controllers



No Adaptation



With Adaptation

- Adaptive VS Non Adaptive
  - Number of control switches: 37 VS 41 (9.8% less actuation)
  - Maneuver time: 13 hr for both since maneuver is stopped after a set time

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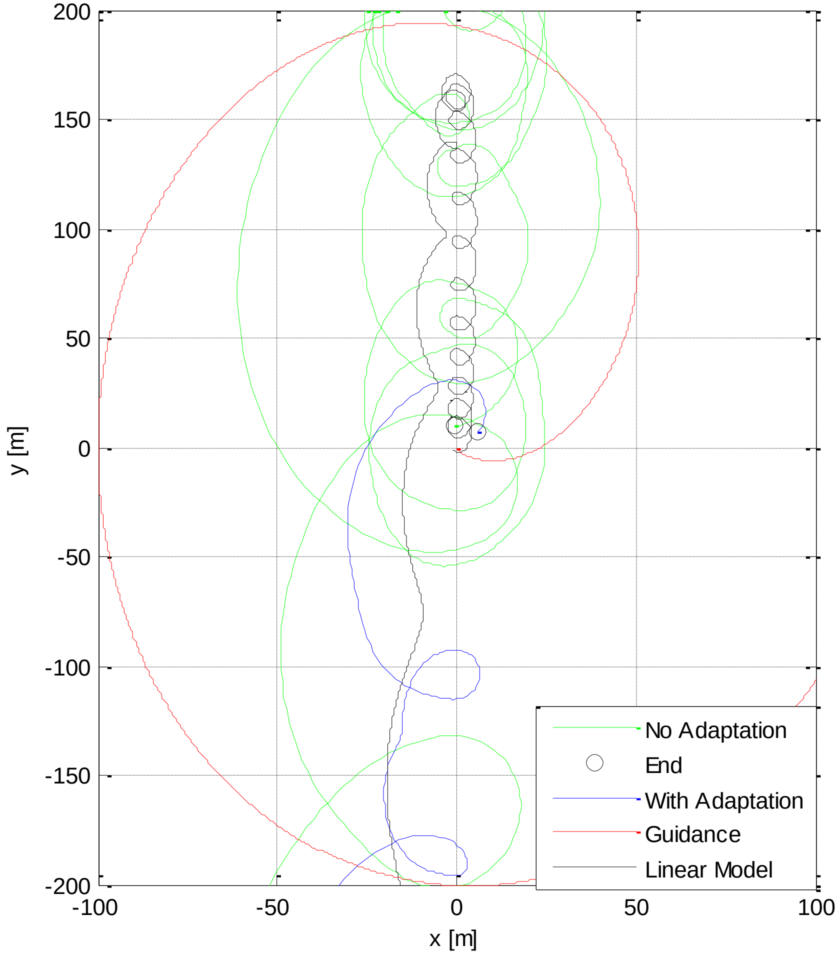
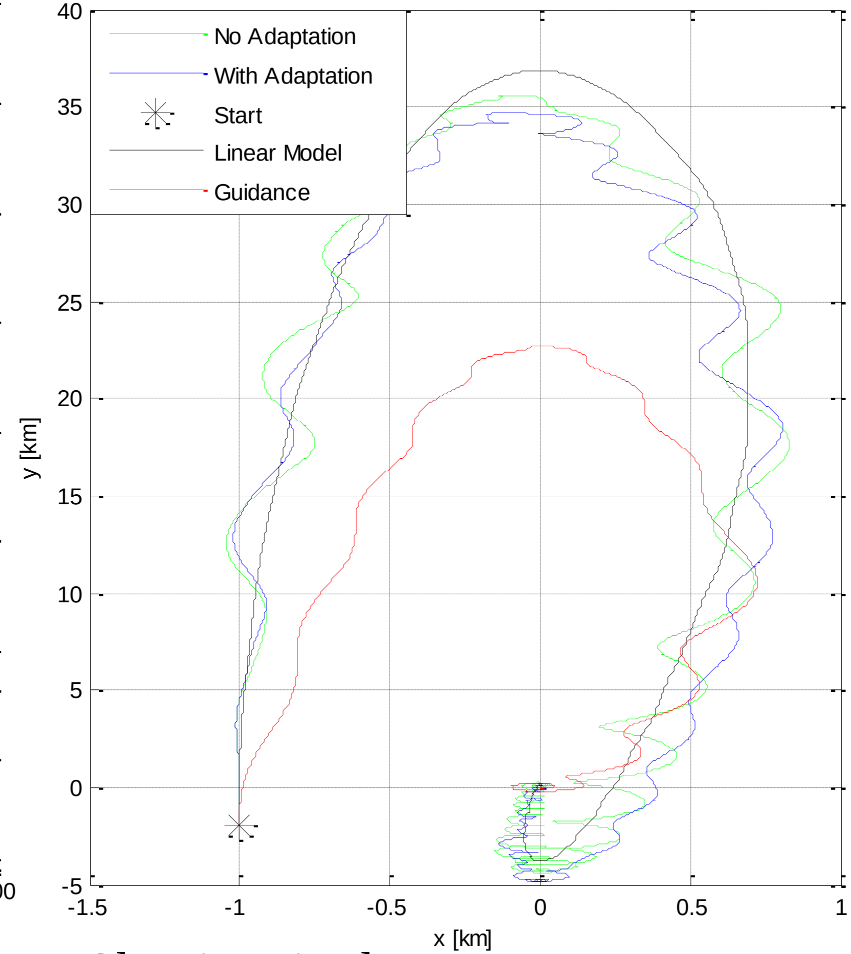
Numerical Simulations: Rendezvous Case 1

- Simulated trajectory in the x-y plane





- Initial relative position of -1km in x, -2km in y in the LVLH
- Less realistic linear reference model for the rendezvous ($R_{LQR}=1.6*10^{18}$)

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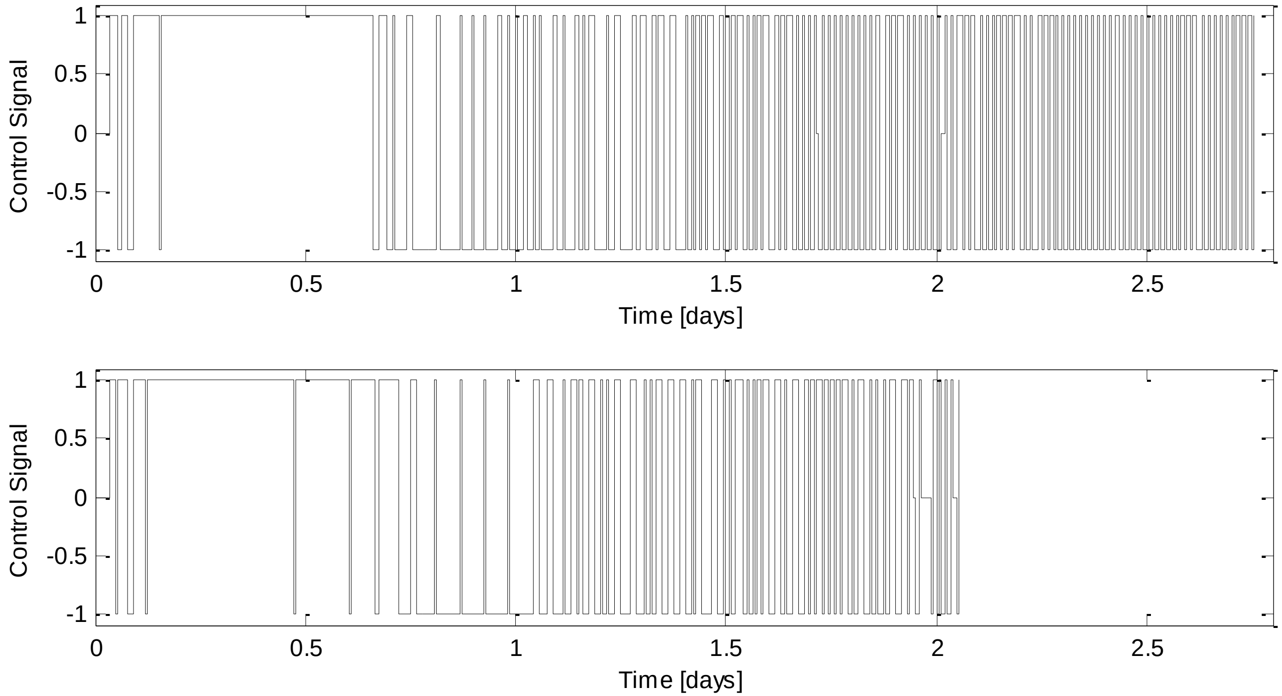
Numerical Simulations: Rendezvous Case 1

- Control signal for both controllers



No Adaptation

With Adaptation

- Adaptive VS Non Adaptive
  - Number of control switches: 124 VS 239 (37% less actuation)
  - Maneuver time: 49 hr VS 66 hr (25.4% less time)

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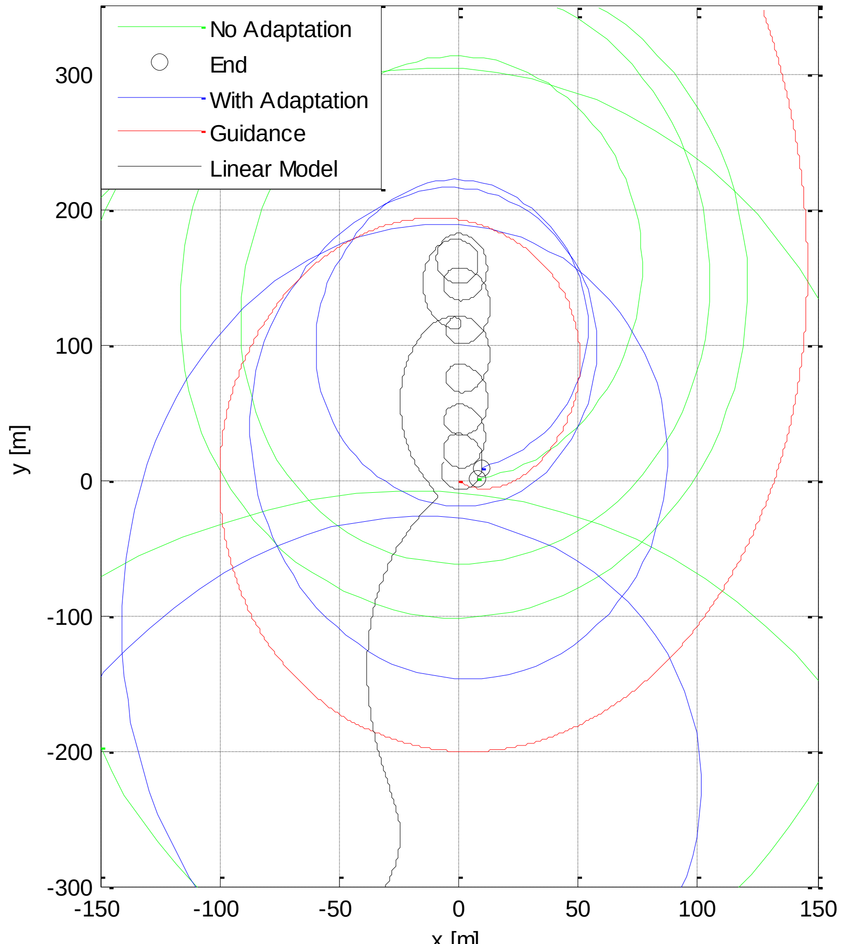
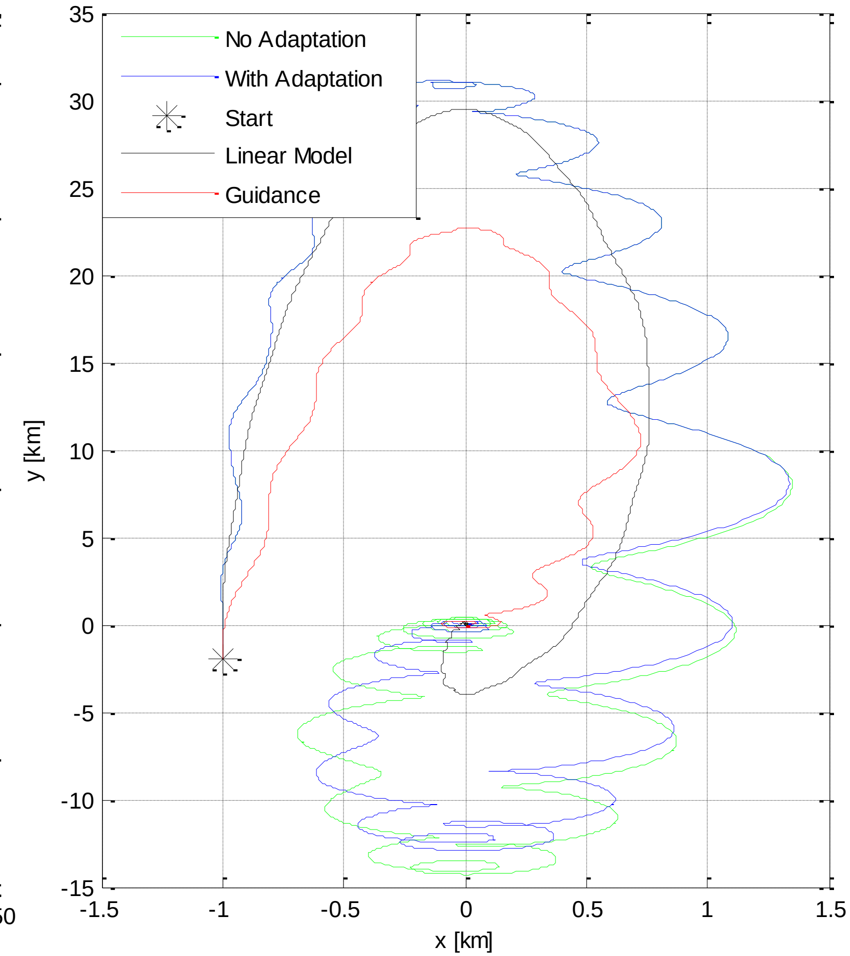
- Simulated trajectory in the x-y plane





- More realistic linear reference model (R_{LQR}=1.5*10^{17}).

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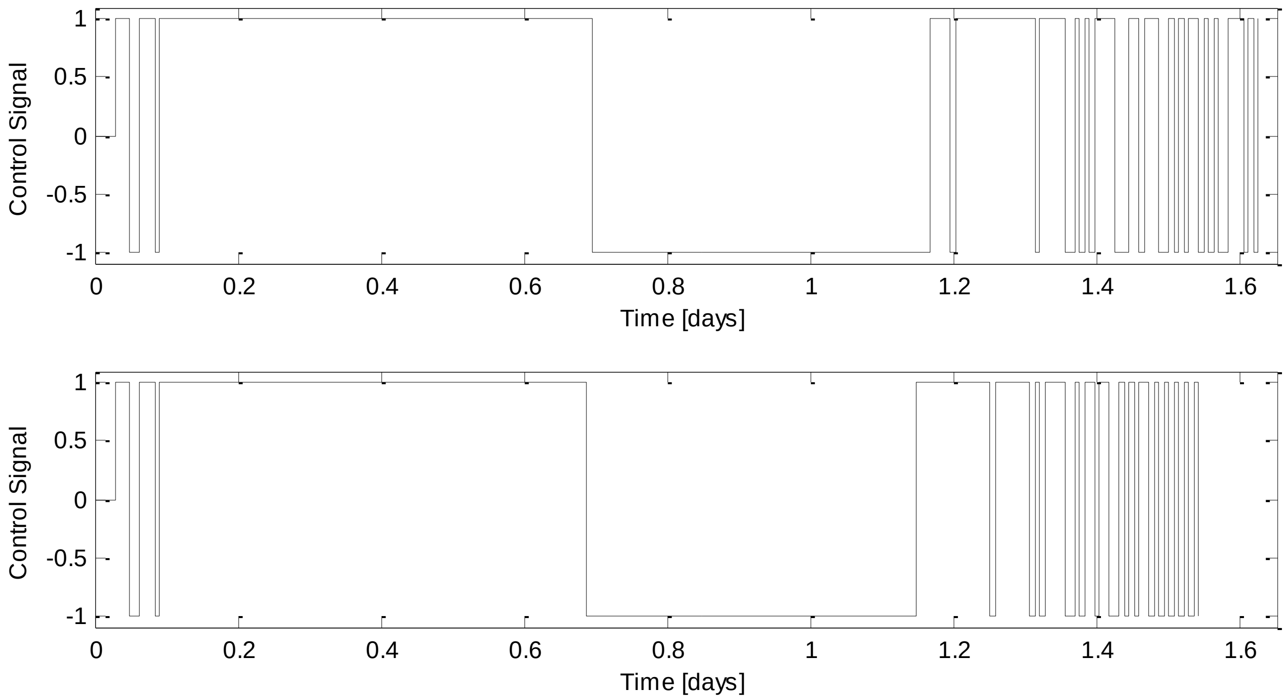
Numerical Simulations: Rendezvous Case 2

- Control signal for both controllers



No Adaptation

With Adaptation

- Adaptive VS Non Adaptive
  - Number of control switches: 36 VS 37 (2.7% less actuation)
  - Maneuver time: 37 hr VS 39 hr (4.9% less time)

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Conclusions

- The adaptive Lyapunov controller enables tracking of a trajectory, the dynamics of a reference model, or simply regulating to a desired final state

- Adaptation provides smoother maneuvers with less duration, less actuation, and greater control margin for the three different controller configurations studied.

- The use of the general derivatives will allow for the implementation of the adaptive Lyapunov controller in maneuvers, in which a specific path is desired, consequently, opening the possibilities for many other maneuvers using differential drag, provided that they are confined to the orbital plane.

- If the linear reference model is not accurate (unrealistic), the adaptive controller is capable of tune itself; thus improving its performance.

- more sophisticated adaptation methods (EIGENVALUES) are expected to significantly improve the ability of the adaptive controller to perform well even when the linear reference model greatly misrepresents the actual dynamics

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