AFOSR Space Propulsion Program Review
September 10, 2012

Propellant-free Spacecraft Relative Maneuvering via Atmospheric Differential Drag

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why not change the world?[SM]

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Outline

- Introduction
- Drag Acceleration
- Linear reference model and Nonlinear Model
- Lyapunov Approach
- Drag devices activation strategy
- Critical value for the magnitude of differential drag acceleration
- Adaptive Lyapunov Control strategy
- Numerical Simulations (3 types of maneuvers)
- Future work on year I
- Accomplishments
- Conclusions

- Years to come…

Introduction

- S/C rendezvous maneuvers are critical for:
  - On-orbit maintenance missions
  - Refueling and autonomous assembly of structures in space
  - Envisioned operations by NASA's Satellite Servicing Capabilities Office
- High cost of refueling calls for an alternative for thrusters as the source of the control forces
- At LEO drag forces are an alternative
- An Adaptive Lyapunov control strategy for the rendezvous maneuver using aerodynamic differential drag is presented

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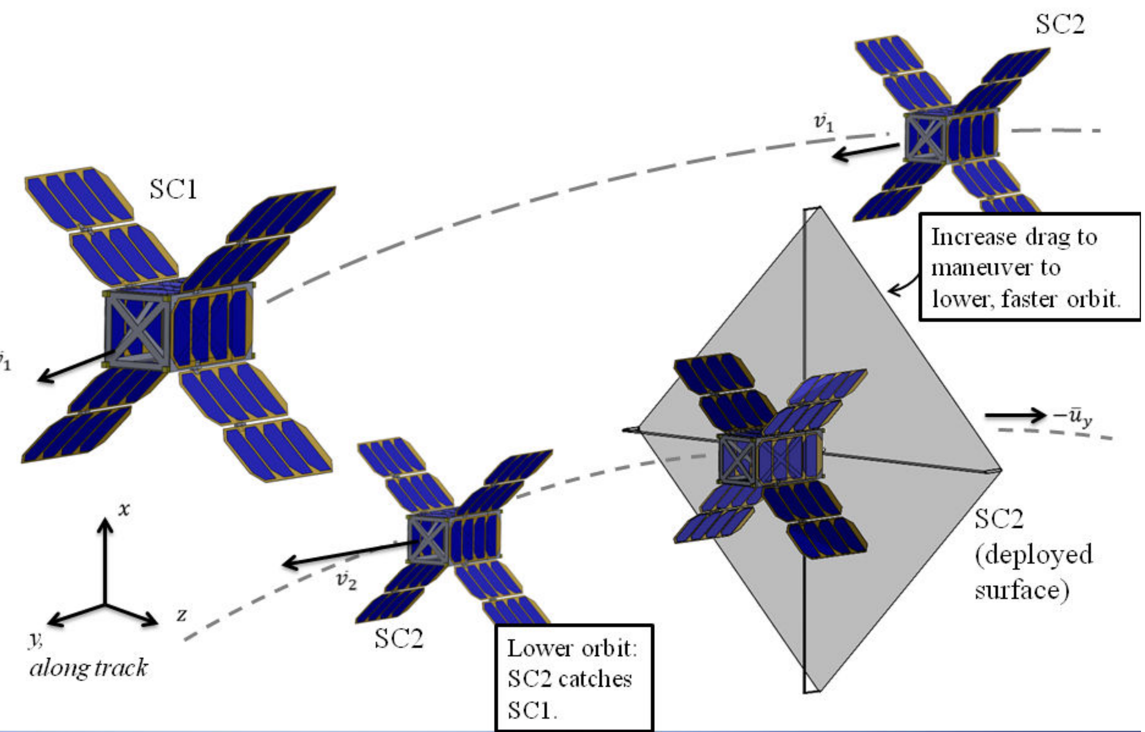
Introduction

- Differential in the aerodynamic drag is a differential in along track acceleration
- This differential can be used to control the relative motion of the S/C on the orbital plane only
- The drag differential can be generated with controllable surfaces
- It is assumed that that the surfaces move almost instantly (on-off control)



Starting References

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- Schweighart, S. A., and Sedwick, R. J., "High-Fidelity Linearized J2 Model for Satellite Formation Flight," Journal of Guidance, Control, and Dynamics, Vol. 25, No. 6, 2002, pp. 1073–1080.
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- Graham, Alexander. Kronecker Products and Matrix Calculus: With Applications. Chichester: Horwood, 1981. Print.
- Bevilacqua, R., Romano, M., "Rendezvous Maneuvers of Multiple Spacecraft by Differential Drag under J2 Perturbation", AIAA Journal of Guidance, Control and Dynamics, vol.31 no.6 (1595-1607), 2008. DOI: 10.2514/1.36362

- Perez, D., Bevilacqua, R., "Differential Drag Spacecraft Rendezvous using an Adaptive Lyapunov Control Strategy", TO APPEAR ON ACTA ASTRONAUTICA.

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Drag Acceleration

- The drag acceleration experienced by a S/C at LEO is a function of:
  - Atmospheric density
  - Atmospheric winds
  - Velocity of the S/C relative to the medium,
  - Geometry, attitude, drag coefficient and mass of the S/C
- Challenges for modeling drag force:
  - The interdependence of these parameters
  - Lack of knowledge in some of their dynamics
- Large uncertainties on the control forces (drag forces)
- Control systems for drag maneuvers must cope with these uncertainties.
- Differential aerodynamic drag for the S/C system is given as:

$$a_{Drel} = \frac{1}{2}\rho \Delta BC v_s^2 \qquad BC = \frac{C_D A}{m}$$

Linear Reference Model

- The Schweighart and Sedwick model is used to create the stable reference model
- LQR controller is used to stabilize the Schweighart and Sedwick model
- The resulting reference model is described by:

$$\dot{\boldsymbol{x}}_d = \underline{\boldsymbol{A}}_d \boldsymbol{x}_d, \quad \underline{\boldsymbol{A}}_d = \underline{\boldsymbol{A}} - \underline{\boldsymbol{B}}\underline{\boldsymbol{K}}, \quad \boldsymbol{x}_d = \begin{bmatrix} x_d & y_d & \dot{x}_d & \dot{y}_d \end{bmatrix}^T$$

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Nonlinear Model

- The dynamics of S/C relative motion are nonlinear due to
  - J_2 perturbation
  - Variations on the atmospheric density at LEO
  - Solar pressure radiation
  - Etc.
- The general expression for the real world nonlinear dynamics, including nonlinearities is:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \underline{\boldsymbol{B}}\boldsymbol{u},\ \boldsymbol{x} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T, \qquad u = \begin{cases} a_{Drel} \\ 0 \\ -a_{Drel} \end{cases}$$

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Lyapunov Approach

- A Lyapunov function of the tracking error is defined as: $V = \boldsymbol{e}^T \underline{\boldsymbol{P}}\boldsymbol{e}, \qquad \boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_d, \qquad \underline{\boldsymbol{P}} \succ 0$
- After some algebraic manipulation, the time derivative of the Lyapunov function is:

$$\dot{V} = \boldsymbol{e}^T (\underline{\boldsymbol{A}}_d{}^T \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_d)\boldsymbol{e} + 2\boldsymbol{e}^T \underline{\boldsymbol{P}}(\boldsymbol{f}(\boldsymbol{x}) - \underline{\boldsymbol{A}}_d \boldsymbol{x} + \underline{\boldsymbol{B}} a_{Drel}\hat{u} - \underline{\boldsymbol{B}}\boldsymbol{u}_d)$$

- Defining $\underline{\boldsymbol{A}}_d$ Hurwitz and $\underline{\boldsymbol{Q}}$ symmetric positive definite, $\underline{\boldsymbol{P}}$ can be found using:

$$-\underline{\boldsymbol{Q}} = \underline{\boldsymbol{A}}_d{}^T \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_d$$

- If the desired guidance is a constant zero state vector (controller acts as a regulator)

$$\dot{V} = 2\boldsymbol{e}^T \underline{\boldsymbol{P}}(\boldsymbol{f}(\boldsymbol{x}) + \underline{\boldsymbol{B}} a_{Drel}\hat{u})$$

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Drag panels activation strategy

- Rearranging $\dot{V}$ yields

$$\dot{V} = 2\left(\beta\hat{u} - \delta\right),$$

$$\beta = \boldsymbol{e}^{T}\underline{\boldsymbol{P}}\underline{\boldsymbol{B}}a_{Drel}, \quad \delta = -\boldsymbol{e}^{T}\underline{\boldsymbol{P}}\,\boldsymbol{f}\left(\boldsymbol{x}\right), \quad \hat{u} = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

- Guaranteeing $\dot{V} < 0$ would imply that the tracking error (***e***) converges to zero
- By selecting:

$$\hat{u} = -sign(\beta) = -sign(\boldsymbol{e}^{T}\underline{\boldsymbol{P}}\underline{\boldsymbol{B}})$$

$\dot{V}$ is ensured to be as small as possible.

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Critical value for the magnitude of differential drag acceleration

- Product $\beta\hat{u}$ is the only controllable term that influences the behavior of $\dot{V} = 2\left(\beta\hat{u} - \delta\right)$
- There must be a minimum value for a_{Drel} that allows for $\dot{V}$ to be negative for given values of β and δ
- This value is found analytically by solving:

$$0 \geq \boldsymbol{e}^T \underline{\boldsymbol{P}}\,\underline{\boldsymbol{B}} a_{Drel} \hat{u} - \delta$$

- Solving this expression for a_{Drel} yields

$$a_{Drel} \geq \frac{\delta}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\,\underline{\boldsymbol{B}}\right|} = \frac{-\boldsymbol{e}^T \underline{\boldsymbol{P}}\, \boldsymbol{f}\left(\boldsymbol{x}\right)}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\,\underline{\boldsymbol{B}}\right|} = a_{Dcrit}$$

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Matrix derivatives

- Choosing appropriate values for the entries of $\underline{\boldsymbol{Q}}$ and $\underline{\boldsymbol{A}}_d$ can reduce a_{Dcrit}
- To achieve this, the following partial derivatives were developed

$$\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{A}}_d}, \quad \frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{Q}}}$$

- The first step to find them is to develop:

$$a_{Dcrit} = \frac{-\boldsymbol{e}^T \underline{\boldsymbol{P}}\, \boldsymbol{f}(\boldsymbol{x})}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|}, \quad \frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{P}}} = \frac{\boldsymbol{e}^T \boldsymbol{f}(\boldsymbol{x})}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|} - \frac{\left(\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right)\left(\boldsymbol{e}^T \underline{\boldsymbol{P}}\, \boldsymbol{f}(\boldsymbol{x})\right)\boldsymbol{e}\underline{\boldsymbol{B}}^T}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|^3}$$

- Afterwards the Lyapunov equation was transformed into:

$$-\underline{\boldsymbol{Q}} = \underline{\boldsymbol{A}}_d^{\,T}\underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_d, \quad \underline{\boldsymbol{A}}_v \boldsymbol{P}_v = -\boldsymbol{Q}_v,$$

$$\underline{\boldsymbol{A}}_v = \underline{\mathbf{I}}_{4x4} \otimes \underline{\boldsymbol{A}}_d + \underline{\boldsymbol{A}}_d \otimes \underline{\mathbf{I}}_{4x4}, \quad \boldsymbol{P}_v = vec(\underline{\boldsymbol{P}}), \quad \boldsymbol{Q}_v = vec(\underline{\boldsymbol{Q}}),$$

$$\boldsymbol{P}_v = -\underline{\boldsymbol{A}}_v^{-1}\boldsymbol{Q}_v$$

Matrix derivatives

- Vec operator and Kronecker product

$$vec(\underline{\mathbf{Z}}) = \mathbf{Z}_v = \begin{bmatrix} Z_{11} & \cdots & Z_{n1} & \cdots & Z_{1n} & \cdots & Z_{nn} \end{bmatrix}^T, \quad \underline{\mathbf{X}} \otimes \underline{\mathbf{Y}} = \begin{bmatrix} (X_{11}\underline{\mathbf{Y}}) & \cdots & (X_{1n}\underline{\mathbf{Y}}) \\ \vdots & \ddots & \vdots \\ (X_{n1}\underline{\mathbf{Y}}) & \cdots & (X_{nn}\underline{\mathbf{Y}}) \end{bmatrix}$$

- Matrix Derivatives

$$\frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \underline{\mathbf{Y}}, \underline{\mathbf{X}} = \begin{bmatrix} (\underline{\mathbf{Y}}, X_{11}) & \cdots & (\underline{\mathbf{Y}}, X_{1n}) \\ \vdots & \ddots & \vdots \\ (\underline{\mathbf{Y}}, X_{n1}) & \cdots & (\underline{\mathbf{Y}}, X_{nn}) \end{bmatrix}, \quad \frac{\partial \mathbf{Y}_v}{\partial \underline{\mathbf{X}}} = vec(\underline{\mathbf{Y}}), \underline{\mathbf{X}} = \begin{bmatrix} (vec(\underline{\mathbf{Y}}), X_{11}) & \cdots & (vec(\underline{\mathbf{Y}}), X_{1n}) \\ \vdots & \ddots & \vdots \\ (vec(\underline{\mathbf{Y}}), X_{n1}) & \cdots & (vec(\underline{\mathbf{Y}}), X_{nn}) \end{bmatrix},$$

$$\frac{\partial \mathbf{Y}_v}{\partial \mathbf{X}_v} = vec(\underline{\mathbf{Y}}), vec(\underline{\mathbf{X}}) = \begin{bmatrix} (vec(\underline{\mathbf{Y}}))^T, X_{11} \\ \vdots \\ (vec(\underline{\mathbf{Y}}))^T, X_{n1} \\ \vdots \\ (vec(\underline{\mathbf{Y}}))^T, X_{1n} \\ \vdots \\ (vec(\underline{\mathbf{Y}}))^T, X_{nn} \end{bmatrix}, \quad \frac{\partial [\mathbf{Y}_v]^T}{\partial \underline{\mathbf{X}}} = [vec(\underline{\mathbf{Y}})]^T, \underline{\mathbf{X}} = \begin{bmatrix} ([vec(\underline{\mathbf{Y}})]^T, X_{11}) & \cdots & ([vec(\underline{\mathbf{Y}})]^T, X_{1n}) \\ \vdots & \ddots & \vdots \\ ([vec(\underline{\mathbf{Y}})]^T, X_{n1}) & \cdots & ([vec(\underline{\mathbf{Y}})]^T, X_{nn}) \end{bmatrix}$$

- Matrix Derivative Transformations

$$\frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \mathrm{T}_1\left(\frac{\partial \mathbf{Y}_v}{\partial \mathbf{X}_v}\right), \quad \frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \mathrm{T}_2\left(\frac{\partial \mathbf{Y}_v}{\partial \underline{\mathbf{X}}}\right), \quad \frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \mathrm{T}_3\left(\frac{\partial [\mathbf{Y}_v]^T}{\partial \underline{\mathbf{X}}}\right)$$

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Matrix derivatives

- Using $\underline{P}_v = -\underline{A}_v^{-1}\underline{Q}_v$

The following derivatives can be found:

$$\frac{\partial \underline{P}}{\partial \underline{Q}} = \mathrm{T}_1\left(\left(-\underline{A}_v^{-1}\right)^T\right), \quad \frac{\partial \underline{P}_v}{\partial \underline{A}_v} = \left(\underline{\mathbf{I}}_{16x16} \otimes \underline{A}_v^{-1}\right)\underline{U}_{16x16}\left(\underline{\mathbf{I}}_{16x16} \otimes \underline{A}_v^{-1}\right)\left(\underline{\mathbf{I}}_{16x16} \otimes \underline{Q}_v\right),$$

$$\frac{\partial \underline{P}}{\partial \underline{A}_d} = \mathrm{T}_2\left(\frac{\partial \underline{P}_v}{\partial \underline{A}_d}\right), \quad \frac{\partial \underline{A}_v}{\partial \underline{A}_d} = \left(\underline{\mathbf{I}}_{4x4} \otimes \underline{U}_1\right)\left(\underline{U}_{4x4} \otimes \underline{\mathbf{I}}_{4x4}\right)\left(\underline{\mathbf{I}}_{4x4} \otimes \underline{U}_1\right) + \underline{U}_{4x4} \otimes \underline{\mathbf{I}}_{4x4}$$

- Using the chain rule the desired final expressions can be found:

$$\frac{\partial a_{Dcrit}}{\partial \underline{Q}} = \mathrm{T}_3^{-1}\left(\frac{\partial \underline{P}}{\partial \underline{Q}}\right)\left[\underline{\mathbf{I}}_{4x4} \otimes \mathrm{T}_1^{-1}\left(\frac{\partial a_{Dcrit}}{\partial \underline{P}}\right)\right],$$

$$\frac{\partial a_{Dcrit}}{\partial \underline{A}_d} = \mathrm{T}_3^{-1}\left(\frac{\partial \underline{P}}{\partial \underline{A}_d}\right)\left[\underline{\mathbf{I}}_{4x4} \otimes \mathrm{T}_1^{-1}\left(\frac{\partial a_{Dcrit}}{\partial \underline{P}}\right)\right]$$

Adaptive Lyapunov Control strategy

- Using these derivatives $\underline{A}_d$ and $\underline{Q}$ are adapted as follows:

$$\frac{dA_{ij}}{dt} = \kappa_A\left[-sign(\frac{\partial a_{Dcrit}}{\partial A_{ij}})\delta_A\right], \quad \frac{dQ_{ij}}{dt} = \kappa_Q\left[-sign(\frac{\partial a_{Dcrit}}{\partial Q_{ij}})\delta_Q\right]$$

$$\kappa_A = \begin{cases} 1 \text{ if } \frac{\partial a_{Dcrit}}{\partial A_{ij}} > \frac{\partial a_{Dcrit}}{\partial A_{kl}} \text{ for } i,j \neq k,l \\ 0 \text{ else} \end{cases}, \quad \kappa_Q = \begin{cases} 1 \text{ if } \frac{\partial a_{Dcrit}}{\partial Q_{ij}} > \frac{\partial a_{Dcrit}}{\partial Q_{kl}} \text{ for } i,j \neq k,l \\ 0 \text{ else} \end{cases}$$

- These were designed such that:
  - $\underline{Q}$ is symmetric positive definite
  - $\underline{A}_d$ is Hurwitz
- These adaptations result in an adaptation of the quadratic Lyapunov function

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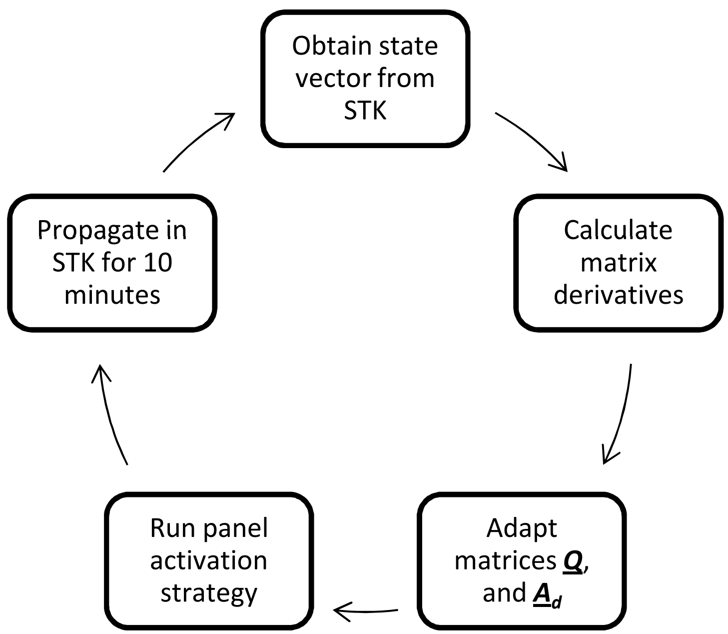
Numerical Simulations

- Simulations were performed using an STK scenario with High-Precision Orbit Propagator (HPOP) that included:
  - Full gravitational field model
  - Variable atmospheric density (using NRLMSISE-00)
  - Solar pressure radiation effects

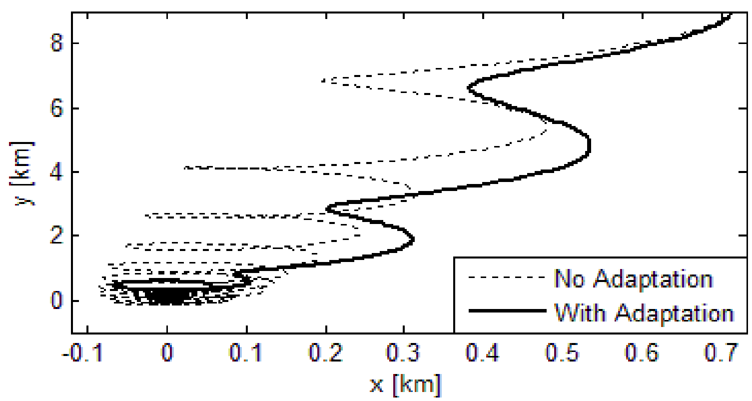


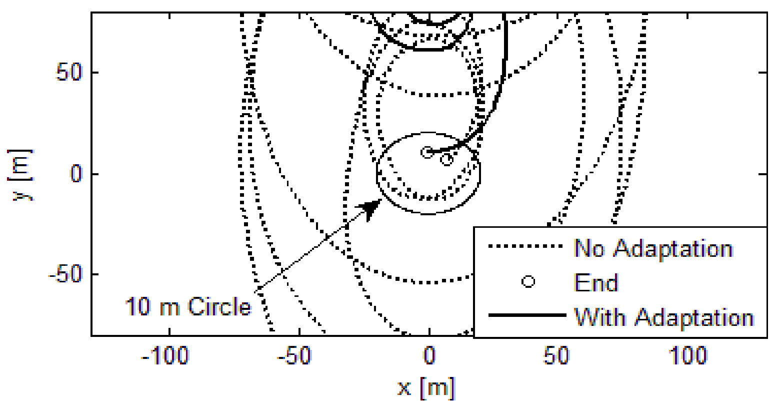
Parameter	Value
Target's inclination (deg)	98
Target's semi-major axis (km)	6778
Target's right ascension of the ascending node (deg)	262
Target's argument of perigee (deg)	30
Target's true anomaly (deg)	25
Target's eccentricity	0
m(kg)	10
$S(m^2)$	1.3
C_D	2

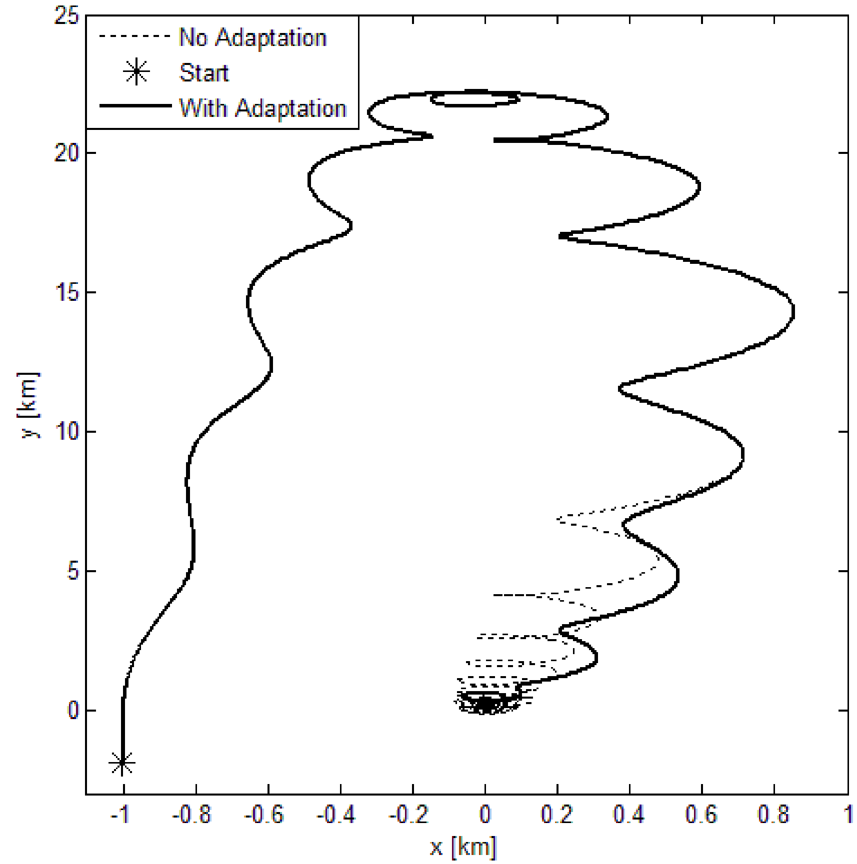
- Initial relative position of -1km in x, -2km in y in the LVLH
- The maneuver ended when S/C were within 10m.

Numerical Simulations (rendezvous)

❑Simulated trajectory in the x-y plane







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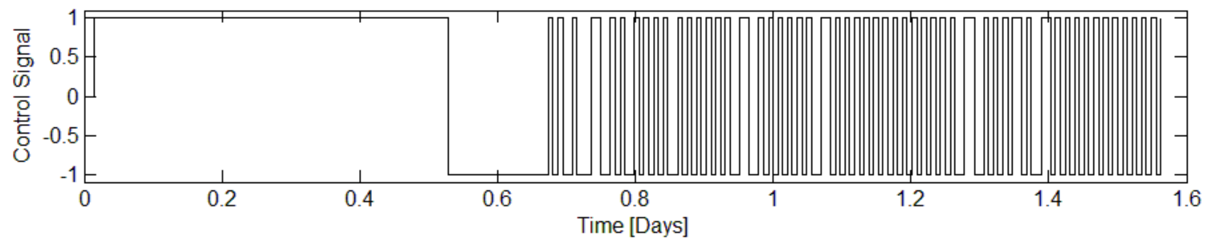
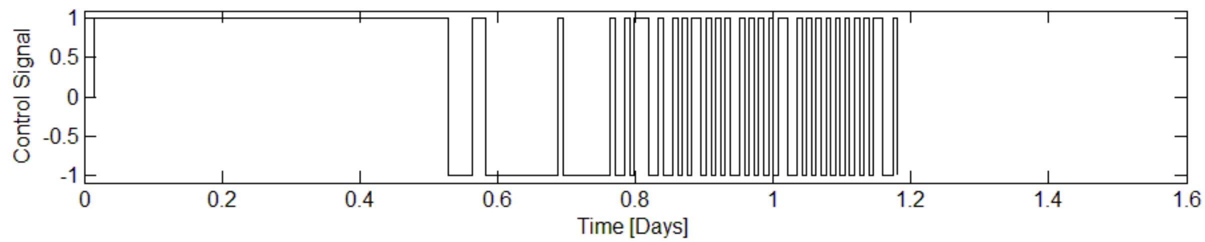
Numerical Simulations (rendezvous)

- Control signal for both controllers

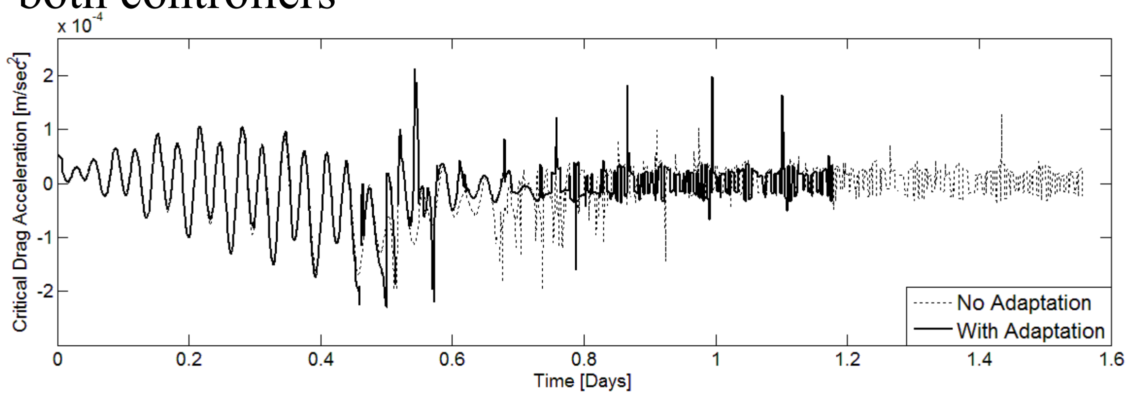


No Adaptation



With Adaptation

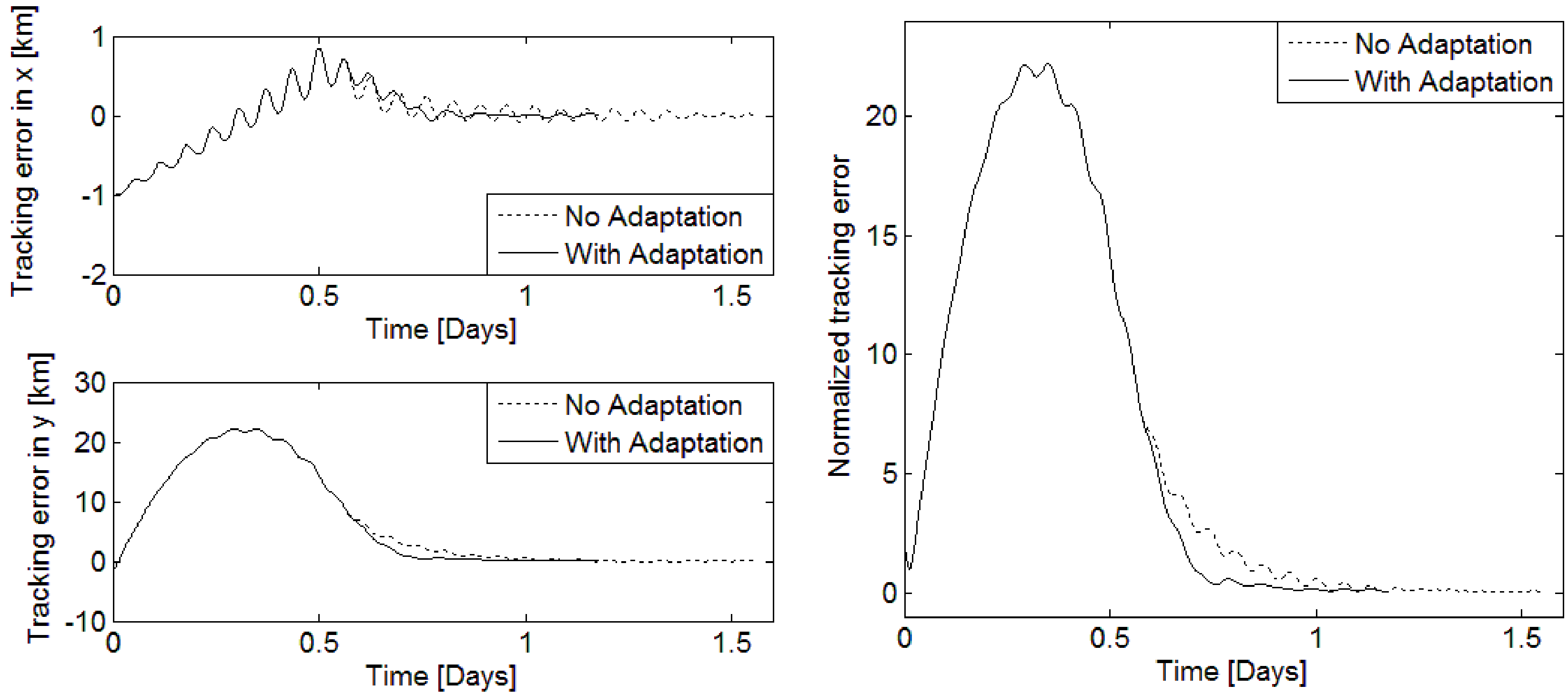
- a_{Dcrit} for both controllers



- Adaptive VS Non Adaptive
  - Number of control switches: 56 VS 113 (50% less actuation)
  - Maneuver time: 29 hr VS 38 hr (24% less time)

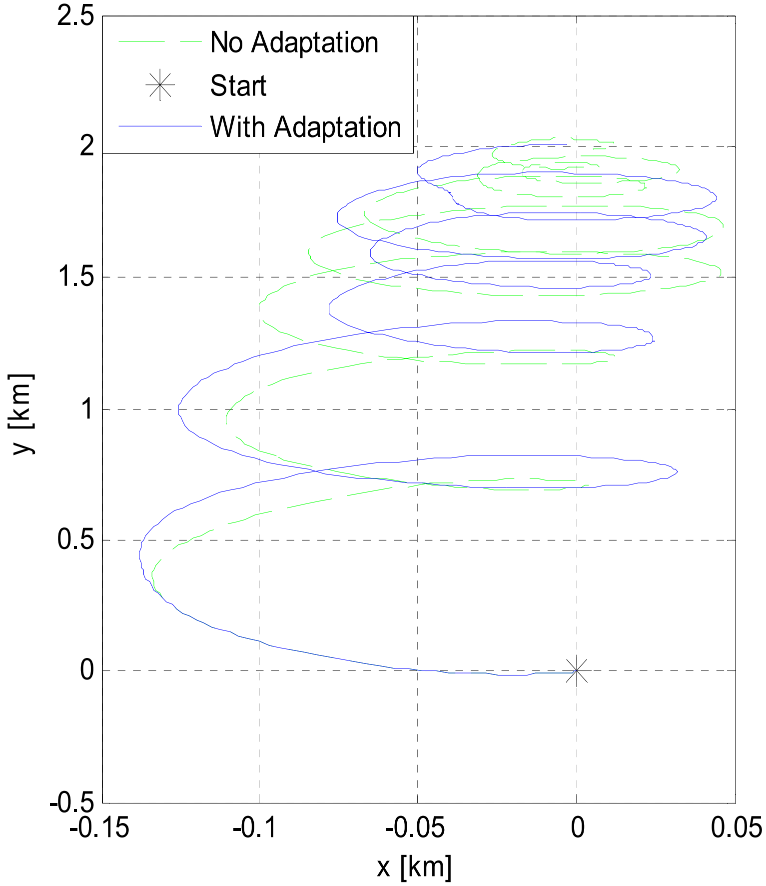
Numerical Simulations (rendezvous)

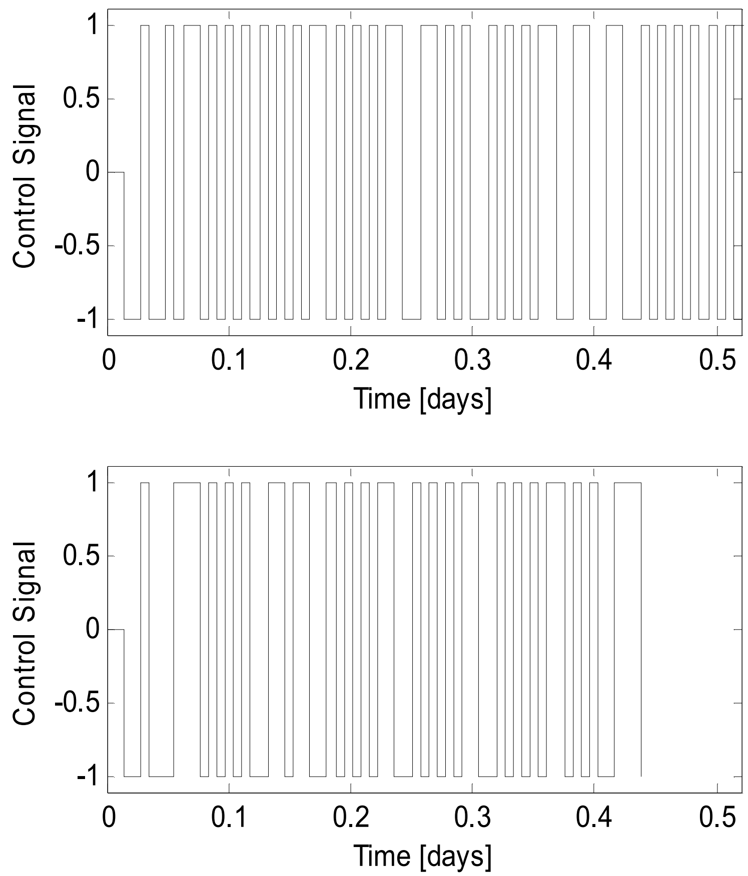
- Error for both controllers



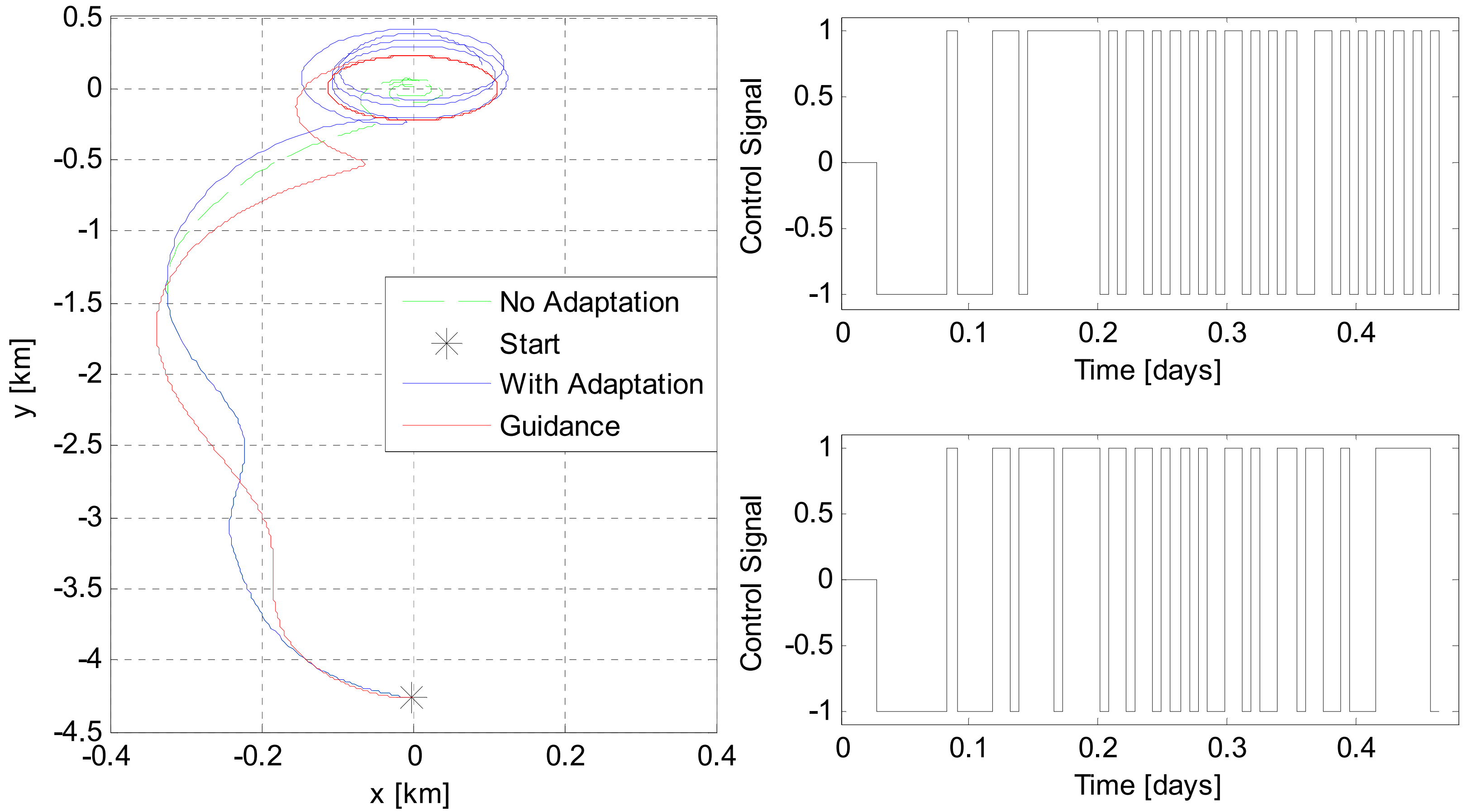
- Non adaptive Lyapunov controller needs more time and a higher control effort since it approaches the rendezvous state performing larger oscillations
- The reduction on the maneuver time and the control effort is caused by the adaptation of the matrix ***P*** which allows the adaptive Lyapunov control to tune itself as the error evolves

Numerical Simulations (re-phasing)





Numerical Simulations (fly-around)



Future Work

- Include the linear reference model in the derivation of: $\frac{\partial a_{Dcrit}}{\partial \underline{A}_d}, \frac{\partial a_{Dcrit}}{\partial \underline{Q}}$
  - This will allow for tracking a desired path or the dynamics of the linear reference model
- Further developments on the adaptation strategy are expected to improve controller performance
- Attitude control via drag too(?)

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Conclusions

- A novel adaptive Lyapunov controller for S/C autonomous rendezvous maneuvers using atmospheric differential drag is presented
- Analytical expressions a_{Dcrit} , $\frac{\partial a_{Dcrit}}{\partial \underline{A}_d}, \frac{\partial a_{Dcrit}}{\partial \underline{Q}}$ are derived
- The quadratic Lyapunov function is modified in real time, during flight using these derivatives, reducing a_{Dcrit}
- Unprecedented ability to perform rendezvous to less than 10 meters without propellant
- Adaptive Lyapunov controller is an improvement
  - Significantly lower control effort (50% less actuation)
  - Less time to reach the desired rendezvous state (24% less time)

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Accomplishments (April-September 2012)

Perez, D., Bevilacqua, R., "Differential Drag Spacecraft Rendezvous using an Adaptive Lyapunov Control Strategy", accepted for publication on Acta Astronautica, to appear.

BEST STUDENT PAPER AWARD FOR THE CATEGORY: SPACECRAFT GUIDANCE, NAVIGATION, AND CONTROL.
1st International Academy of Astronautics Conference on Dynamics and Control of Space Systems – DyCoSS'2012, Porto, Portugal, 19-21 March 2012.

BEST ORAL PRESENTATION in the Theoretical Category:
Undergraduate research by Skyler Kleinschmidt at the Rensselaer's Third Annual Undergraduate Research Symposium: "Origami-Based Drag Sail for Differential Drag Controlled Satellites", Wednesday, April 4, 2012.

Submitted NSF proposal using differential drag, in collaboration with NASA scientist

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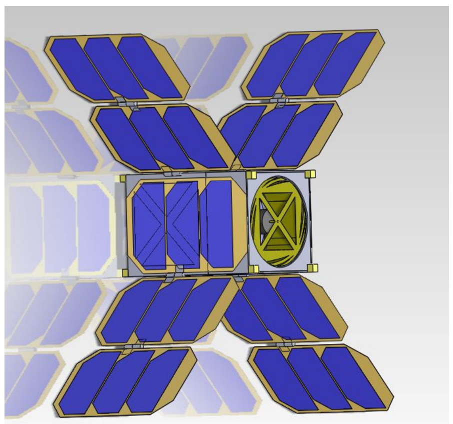
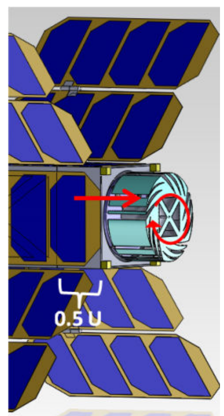
Year II

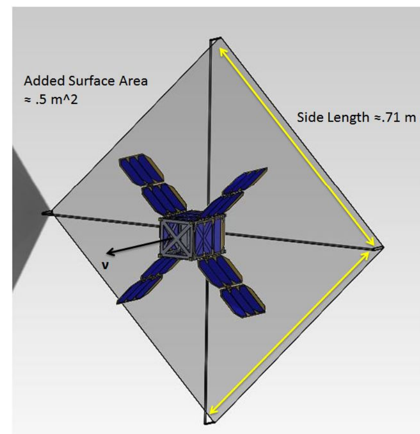
- Study a new methodology to estimate drag and reconstruct neutral density
- Repeat simulations using GRACE (Gravity Recovery and Climate Experiment) data. This data was kindly provided at AFRL Kirtland AFB, during AF SFFP (summer 2012)
- Forecasting on given orbit possible?

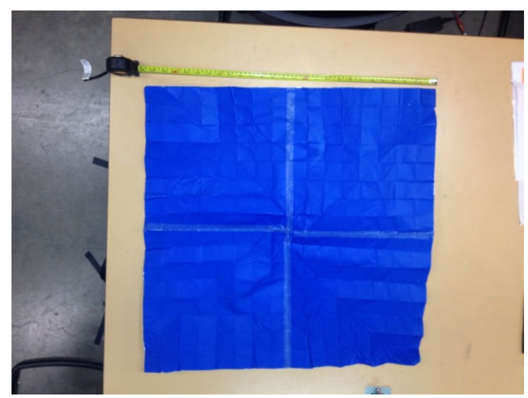
Year III

- 2U Cubesat, hosting a deployable sail in 0.5U
- Clear mylar used for the sail (space qualified, and letting sun get through)
- Sail design based on an origami folding pattern, to maximize surface, and minimize volume when contracted









Questions