1st IAA Conference on Dynamics and Control of Space Systems
March 20th 2012

Differential Drag Spacecraft Rendezvous using an Adaptive Lyapunov Control Strategy

AFOSR
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

ADAMUS

David Pérez
Riccardo Bevilacqua
ADvanced Autonomous MUltiple Spacecraft LAB
Rensselaer Polytechnic Institute

Outline

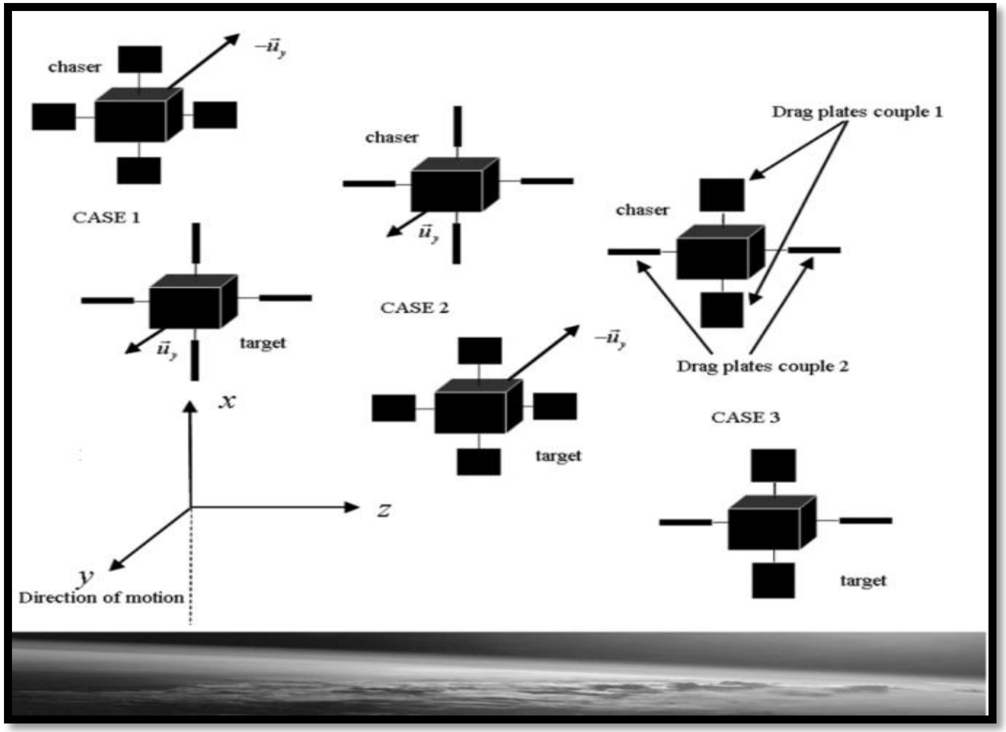
- Introduction
- Drag Acceleration
- Linear reference model and Nonlinear Model
- Lyapunov Approach
- Drag panels activation strategy
- Critical value for the magnitude of differential drag acceleration
- Adaptive Lyapunov Control strategy
- Numerical Simulations

Introduction

- S/C rendezvous maneuvers are critical for:
  - On-orbit maintenance missions
  - Refueling and autonomous assembly of structures in space
  - Envisioned operations by NASA's Satellite Servicing Capabilities Office
- High cost of refueling calls for an alternative for thrusters as the source of the control forces
- At LEO drag forces are an alternative
- An Adaptive Lyapunov control strategy for the rendezvous maneuver using aerodynamic differential drag is presented

Introduction

- Differential in the aerodynamic drag produces a differential in acceleration
- This differential can be used to control the relative motion of the S/C on the orbital plane only
- One possibility to generate the drag differential is to use rotating flat panels
- It is assumed that that the panels rotate almost instantly (on-off control)
- Three cases for the configurations of the panels are considered:



- The foremost contributions in this work are:
  - An analytical expression for a_{Dcrit} .
  - Analytical expressions for :

$$\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{A}}_d} \quad \frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{Q}}}$$

  - Adaptive Lyapunov Control strategy
    - Uses adaptation to choose in real time an appropriate positive definite matrix $\underline{\boldsymbol{P}}$ in a quadratic Lyapunov function such that a_{Dcrit} is reduced on the fly.
    - Does not require numerical iterations
    - Runs in real time, requiring onboard measurements that would be available during flight.
  - Assessment of the approach performances via STK simulations in terms of:
    - Duration of the rendezvous maneuver and the
    - Number of switches in the differential drag (control effort)

Starting References

- Leonard, C. L., Hollister, W., M., and Bergmann, E. V. "Orbital Formationkeeping with Differential Drag". AIAA Journal of Guidance, Control and Dynamics, Vol. 12 (1) (1989), pp.108–113.
- Schweighart, S. A., and Sedwick, R. J., "High-Fidelity Linearized J2 Model for Satellite Formation Flight," Journal of Guidance, Control, and Dynamics, Vol. 25, No. 6, 2002, pp. 1073–1080.
- Curti, F., Romano, M., Bevilacqua, R., "Lyapunov-Based Thrusters' Selection for Spacecraft Control: Analysis and Experimentation", AIAA Journal of Guidance, Control and Dynamics, Vol. 33, No. 4, July–August 2010, pp. 1143-1160. DOI: 10.2514/1.47296.
- Graham, Alexander. Kronecker Products and Matrix Calculus: With Applications. Chichester: Horwood, 1981. Print.
- Bevilacqua, R., Romano, M., "Rendezvous Maneuvers of Multiple Spacecraft by Differential Drag under J2 Perturbation", AIAA Journal of Guidance, Control and Dynamics, vol.31 no.6 (1595-1607), 2008. DOI: 10.2514/1.36362

Drag Acceleration

- The drag acceleration experienced by a S/C at LEO is a function of:
  - Atmospheric density
  - Atmospheric winds
  - Velocity of the S/C relative to the medium,
  - Geometry, attitude, drag coefficient and mass of the S/C
- Challenges for modeling drag force:
  - The interdependence of these parameters
  - Lack of knowledge in some of their dynamics
- Large uncertainties on the control forces (drag forces)
- Control systems for drag maneuvers must cope with these uncertainties.
- Differential aerodynamic drag for the S/C system is given as:

$$a_{Drel} = \frac{1}{2}\rho \Delta BC v_s^2 \qquad BC = \frac{C_D A}{m}$$

- The Schweighart and Sedwick model is used to create the stable reference model
- LQR controller is used to stabilize the Schweighart and Sedwick model
- The resulting reference model is described by:

$$\dot{\boldsymbol{x}}_d = \underline{\boldsymbol{A}}_d \boldsymbol{x}_d, \quad \underline{\boldsymbol{A}}_d = \underline{\boldsymbol{A}} - \underline{\boldsymbol{B}}\underline{\boldsymbol{K}}, \quad \boldsymbol{x}_d = \begin{bmatrix} x_d & y_d & \dot{x}_d & \dot{y}_d \end{bmatrix}^T$$

- $\underline{\boldsymbol{K}}$ is found by solving the LQR problem

Nonlinear Model

- The dynamics of S/C relative motion are nonlinear due to
  - J_2 perturbation
  - Variations on the atmospheric density at LEO
  - Solar pressure radiation
  - Etc.
- The general expression for the real world nonlinear dynamics, including nonlinearities is:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}\right) + \underline{\boldsymbol{B}}\boldsymbol{u}, \quad \boldsymbol{x} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T, \qquad u = \begin{cases} a_{Drel} \\ 0 \\ -a_{Drel} \end{cases}$$

AFOSR AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Rensselaer

David Pérez

- A Lyapunov function of the tracking error is defined as: $V = \boldsymbol{e}^T \underline{\boldsymbol{P}}\boldsymbol{e}, \qquad \boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_d, \qquad \underline{\boldsymbol{P}} \succ 0$
- After some algebraic manipulation, the time derivative of the Lyapunov function is:

$$\dot{V} = \boldsymbol{e}^T (\underline{\boldsymbol{A}}_{\boldsymbol{d}}{}^T \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_{\boldsymbol{d}})\boldsymbol{e} + 2\boldsymbol{e}^T \underline{\boldsymbol{P}}(\boldsymbol{f}(\boldsymbol{x}) - \underline{\boldsymbol{A}}_{\boldsymbol{d}}\boldsymbol{x} + \underline{\boldsymbol{B}}a_{Drel}\hat{u} - \underline{\boldsymbol{B}}\boldsymbol{u}_{\boldsymbol{d}})$$

- Defining $\underline{\boldsymbol{A}}_{\boldsymbol{d}}$ Hurwitz and $\underline{\boldsymbol{Q}}$ symmetric positive definite, $\underline{\boldsymbol{P}}$ can be found using:

$$-\underline{\boldsymbol{Q}} = \underline{\boldsymbol{A}}_{\boldsymbol{d}}{}^T \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_{\boldsymbol{d}}$$

- If the desired guidance is a constant zero state vector (controller acts as a regulator)

$$\dot{V} = 2\boldsymbol{e}^T \underline{\boldsymbol{P}}(\boldsymbol{f}(\boldsymbol{x}) + \underline{\boldsymbol{B}}a_{Drel}\hat{u})$$

- Rearranging $\dot{V}$ yields

$$\dot{V} = 2\left(\beta\hat{u} - \delta\right),$$

$$\beta = \boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}} a_{Drel}, \quad \delta = \text{-}\boldsymbol{e}^T \underline{\boldsymbol{P}}\ \boldsymbol{f}\left(\boldsymbol{x}\right), \quad \hat{u} = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

- Guaranteeing $\dot{V} < 0$ would imply that the tracking error ($\boldsymbol{e}$) converges to zero
- By selecting:

$$\hat{u} = -sign(\beta) = -sign(\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}})$$

$\dot{V}$ is ensured to be as small as possible.

- Product $\beta\hat{u}$ is the only controllable term that influences the behavior of $\dot{V} = 2(\beta\hat{u} - \delta)$
- There must be a minimum value for a_{Drel} that allows for $\dot{V}$ to be negative for given values of β and δ
- This value is found analytically by solving:

$$0 \geq \boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}} a_{Drel} \hat{u} - \delta$$

- Solving this expression for a_{Drel} yields

$$a_{Drel} \geq \frac{\delta}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|} = \frac{-\boldsymbol{e}^T \underline{\boldsymbol{P}}\ \boldsymbol{f}(\boldsymbol{x})}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|} = a_{Dcrit}$$

AFOSR AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Rensselaer

David Pérez

- Choosing appropriate values for the entries of $\underline{\boldsymbol{Q}}$ and $\underline{\boldsymbol{A}}_d$ can minimize a_{Dcrit}
- To achieve this, the following partial derivatives were developed $$\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{A}}_d}, \quad \frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{Q}}}$$
- The first step to find them is to develop:

$$a_{Dcrit} = \frac{-\boldsymbol{e}^T \underline{\boldsymbol{P}}\ \boldsymbol{f}(\boldsymbol{x})}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|}, \qquad \frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{P}}} = \frac{\boldsymbol{e}^T \boldsymbol{f}(\boldsymbol{x})}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|} - \frac{\left(\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right)\left(\boldsymbol{e}^T \underline{\boldsymbol{P}}\ \boldsymbol{f}(\boldsymbol{x})\right)\boldsymbol{e}\underline{\boldsymbol{B}}^T}{\left|\boldsymbol{e}^T \underline{\boldsymbol{P}}\underline{\boldsymbol{B}}\right|^3}$$

- Afterwards the Lyapunov equation was transformed into:

$$-\underline{\boldsymbol{Q}} = \underline{\boldsymbol{A}}_d^{\ T} \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_d, \quad \underline{\boldsymbol{A}}_v \boldsymbol{P}_v = -\boldsymbol{Q}_v,$$

$$\underline{\boldsymbol{A}}_v = \underline{\mathbf{I}}_{4x4} \otimes \underline{\boldsymbol{A}}_d + \underline{\boldsymbol{A}}_d \otimes \underline{\mathbf{I}}_{4x4}, \quad \boldsymbol{P}_v = vec(\underline{\boldsymbol{P}}), \quad \boldsymbol{Q}_v = vec(\underline{\boldsymbol{Q}}),$$

$$\boldsymbol{P}_v = -\underline{\boldsymbol{A}}_v^{-1} \boldsymbol{Q}_v$$

- Using $\mathbf{P}_v = -\underline{\mathbf{A}}_v^{-1}\mathbf{Q}_v$

The following derivatives can be found:

$$\frac{\partial \underline{\mathbf{P}}}{\partial \underline{\mathbf{Q}}} = \mathrm{T}_1\left(\left(-\underline{\mathbf{A}}_v^{-1}\right)^T\right), \quad \frac{\partial \mathbf{P}_v}{\partial \underline{\mathbf{A}}_v} = \left(\underline{\mathbf{I}}_{16x16} \otimes \underline{\mathbf{A}}_v^{-1}\right)\underline{\mathbf{U}}_{16x16}\left(\underline{\mathbf{I}}_{16x16} \otimes \underline{\mathbf{A}}_v^{-1}\right)\left(\underline{\mathbf{I}}_{16x16} \otimes \mathbf{Q}_v\right),$$

$$\frac{\partial \underline{\mathbf{P}}}{\partial \underline{\mathbf{A}}_d} = \mathrm{T}_2\left(\frac{\partial \mathbf{P}_v}{\partial \underline{\mathbf{A}}_d}\right), \quad \frac{\partial \underline{\mathbf{A}}_v}{\partial \underline{\mathbf{A}}_d} = \left(\underline{\mathbf{I}}_{4x4} \otimes \underline{\mathbf{U}}_1\right)\left(\underline{\mathbf{U}}_{4x4} \otimes \underline{\mathbf{I}}_{4x4}\right)\left(\underline{\mathbf{I}}_{4x4} \otimes \underline{\mathbf{U}}_1\right) + \underline{\mathbf{U}}_{4x4} \otimes \underline{\mathbf{I}}_{4x4}$$

- Using the chain rule the desired final expressions can be found:

$$\frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{Q}}} = \mathrm{T}_3^{-1}\left(\frac{\partial \underline{\mathbf{P}}}{\partial \underline{\mathbf{Q}}}\right)\left[\underline{\mathbf{I}}_{4x4} \otimes \mathrm{T}_1^{-1}\left(\frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{P}}}\right)\right],$$

$$\frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{A}}_d} = \mathrm{T}_3^{-1}\left(\frac{\partial \underline{\mathbf{P}}}{\partial \underline{\mathbf{A}}_d}\right)\left[\underline{\mathbf{I}}_{4x4} \otimes \mathrm{T}_1^{-1}\left(\frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{P}}}\right)\right]$$

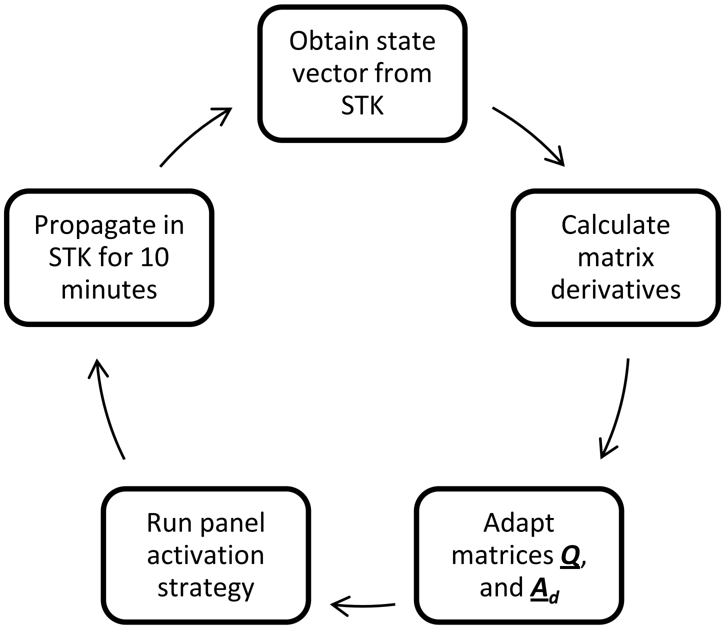
- Using this derivatives $\underline{A_d}$ and $\underline{Q}$ are adapted as follows:

$$\frac{dA_{ij}}{dt} = \kappa_A \left[-sign(\frac{\partial a_{Dcrit}}{\partial A_{ij}})\delta_A \right], \quad \frac{dQ_{ij}}{dt} = \kappa_Q \left[-sign(\frac{\partial a_{Dcrit}}{\partial Q_{ij}})\delta_Q \right]$$

$$\kappa_A = \begin{cases} 1 \text{ if } \dfrac{\partial a_{Dcrit}}{\partial A_{ij}} > \dfrac{\partial a_{Dcrit}}{\partial A_{kl}} \text{ for } i, j \neq k, l \\ 0 \text{ else} \end{cases}, \quad \kappa_Q = \begin{cases} 1 \text{ if } \dfrac{\partial a_{Dcrit}}{\partial Q_{ij}} > \dfrac{\partial a_{Dcrit}}{\partial Q_{kl}} \text{ for } i, j \neq k, l \\ 0 \text{ else} \end{cases}$$

- These were designed such that:
  - $\underline{Q}$ is symmetric positive definite
  - $\underline{A_d}$ is Hurwitz
- These adaptations result in an adaptation of the quadratic Lyapunov function

- Simulations were performed using an STK scenario with High-Precision Orbit Propagator (HPOP) that included:
  - Full gravitational field model
  - Variable atmospheric density (using NRLMSISE-00)
  - Solar pressure radiation effects



Parameter	Value
Target's inclination (deg)	98
Target's semi-major axis (km)	6778
Target's right ascension of the ascending node (deg)	262
Target's argument of perigee (deg)	30
Target's true anomaly (deg)	25
Target's eccentricity	0
m(kg)	10
S(m^2)	1.3
C_D	2

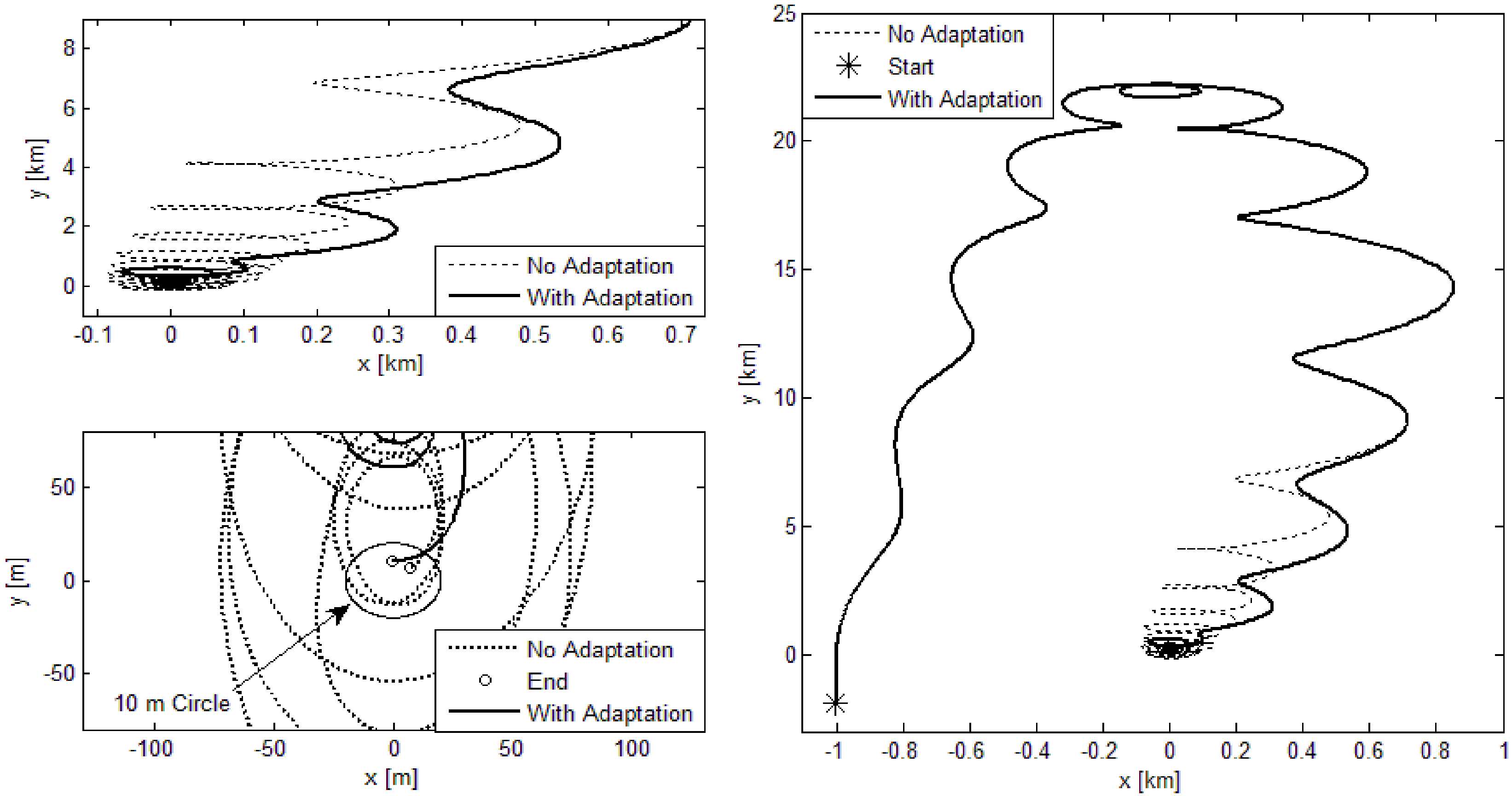
- Initial relative position of -1km in x, -2km in y in the LVLH
- The maneuver ended when S/C were within 10m.

AFOSR AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

David Pérez

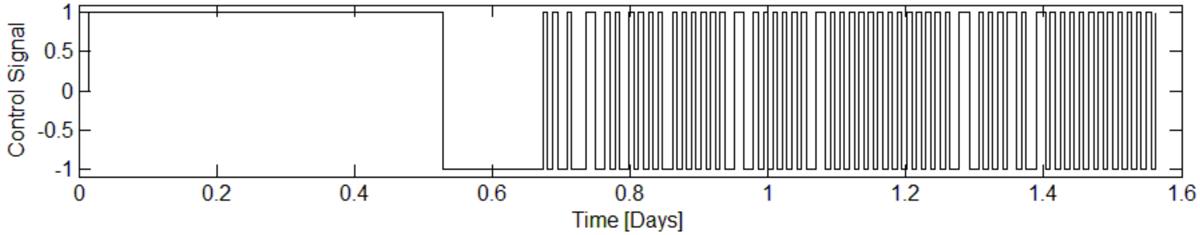
Rensselaer

❑Simulated trajectory in the x-y plane

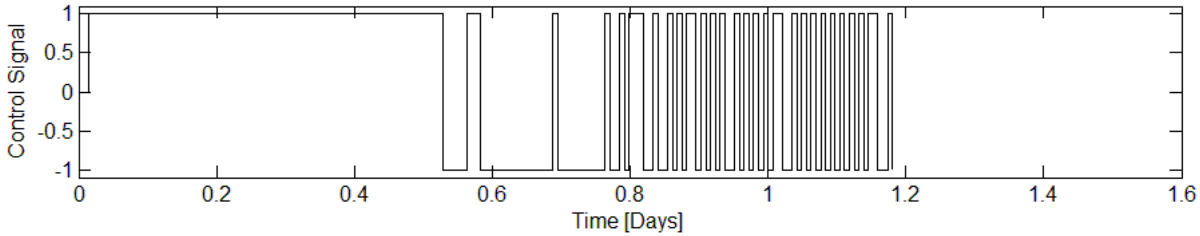


Numerical Simulations

- Control signal for both controllers

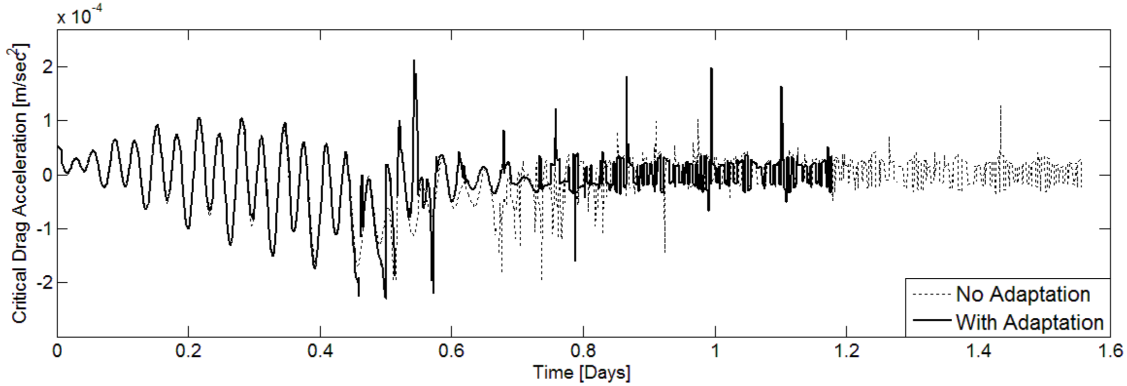


No Adaptation



With Adaptation

- a_{Dcrit} for both controllers



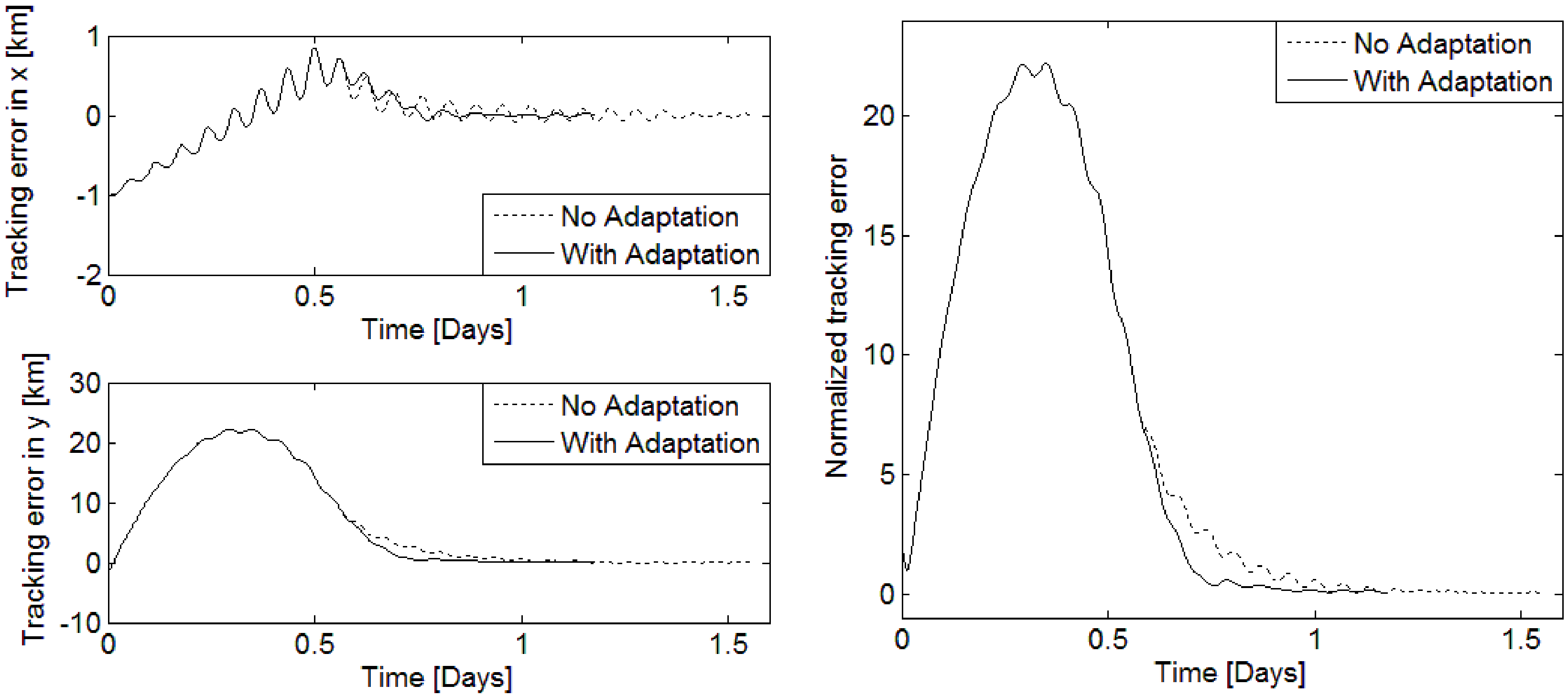
- Adaptive VS Non Adaptive
  - Number of control switches: 56 VS 113 (50% less actuation)
  - Maneuver time: 29 hr VS 38 hr (24% less time)

AFOSR
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

David Pérez

Rensselaer

- Error for both controllers



- Non adaptive Lyapunov controller needs more time and a higher control effort since it approaches the rendezvous state performing larger oscillations
- The reduction on the maneuver time and the control effort is caused by the adaptation of the matrix ***P*** which allows the adaptive Lyapunov control to tune itself as the error evolves

Future Work

- Include the linear reference model in the derivation of: $\frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{A}}_d}, \frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{Q}}}$
  - This will allow for tracking a desired path or the dynamics of the linear reference model
- Further developments on the adaptation strategy are expected to improve controller performance

Conclusions

- A novel adaptive Lyapunov controller for S/C autonomous rendezvous maneuvers using atmospheric differential drag is presented.
- Analytical expressions a_{Dcrit} , $\frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{A}}_d}, \frac{\partial a_{Dcrit}}{\partial \underline{\mathbf{Q}}}$ are derived
- The quadratic Lyapunov function is modified in real time, during flight using these derivatives, minimizing a_{Dcrit} , thus maximizing the control authority margin.
- In simulations both Lyapunov controllers have the unprecedented ability to perform rendezvous to less than 10 meters without propellant
- The resulting behavior of the adaptive Lyapunov controller is an improvement
  - Significantly lower control effort (50% less actuation)
  - Less time to reach the desired rendezvous state (24% less time)

Questions

- Vec operator and Kronecker product

$$vec(\underline{\mathbf{Z}}) = \mathbf{Z}_v = \begin{bmatrix} Z_{11} & \cdots & Z_{n1} & \cdots & Z_{1n} & \cdots & Z_{nn} \end{bmatrix}^T, \quad \underline{\mathbf{X}} \otimes \underline{\mathbf{Y}} = \begin{bmatrix} (X_{11}\underline{\mathbf{Y}}) & \cdots & (X_{1n}\underline{\mathbf{Y}}) \\ \vdots & \ddots & \vdots \\ (X_{n1}\underline{\mathbf{Y}}) & \cdots & (X_{nn}\underline{\mathbf{Y}}) \end{bmatrix}$$

- Matrix Derivatives

$$\frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \underline{\mathbf{Y}}, \underline{\mathbf{X}} = \begin{bmatrix} (\underline{\mathbf{Y}}, X_{11}) & \cdots & (\underline{\mathbf{Y}}, X_{1n}) \\ \vdots & \ddots & \vdots \\ (\underline{\mathbf{Y}}, X_{n1}) & \cdots & (\underline{\mathbf{Y}}, X_{nn}) \end{bmatrix}, \quad \frac{\partial \mathbf{Y}_v}{\partial \underline{\mathbf{X}}} = vec(\underline{\mathbf{Y}}), \underline{\mathbf{X}} = \begin{bmatrix} (vec(\underline{\mathbf{Y}}), X_{11}) & \cdots & (vec(\underline{\mathbf{Y}}), X_{1n}) \\ \vdots & \ddots & \vdots \\ (vec(\underline{\mathbf{Y}}), X_{n1}) & \cdots & (vec(\underline{\mathbf{Y}}), X_{nn}) \end{bmatrix},$$

$$\frac{\partial \mathbf{Y}_v}{\partial \mathbf{X}_v} = vec(\underline{\mathbf{Y}}), vec(\underline{\mathbf{X}}) = \begin{bmatrix} (vec(\underline{\mathbf{Y}}))^T, X_{11} \\ \vdots \\ (vec(\underline{\mathbf{Y}}))^T, X_{n1} \\ \vdots \\ (vec(\underline{\mathbf{Y}}))^T, X_{1n} \\ \vdots \\ (vec(\underline{\mathbf{Y}}))^T, X_{nn} \end{bmatrix}, \quad \frac{\partial [\mathbf{Y}_v]^T}{\partial \underline{\mathbf{X}}} = [vec(\underline{\mathbf{Y}})]^T, \underline{\mathbf{X}} = \begin{bmatrix} ([vec(\underline{\mathbf{Y}})]^T, X_{11}) & \cdots & ([vec(\underline{\mathbf{Y}})]^T, X_{1n}) \\ \vdots & \ddots & \vdots \\ ([vec(\underline{\mathbf{Y}})]^T, X_{n1}) & \cdots & ([vec(\underline{\mathbf{Y}})]^T, X_{nn}) \end{bmatrix}$$

- Matrix Derivative Transformations

$$\frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \mathrm{T}_1\left(\frac{\partial \mathbf{Y}_v}{\partial \mathbf{X}_v}\right), \quad \frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \mathrm{T}_2\left(\frac{\partial \mathbf{Y}_v}{\partial \underline{\mathbf{X}}}\right), \quad \frac{\partial \underline{\mathbf{Y}}}{\partial \underline{\mathbf{X}}} = \mathrm{T}_3\left(\frac{\partial [\mathbf{Y}_v]^T}{\partial \underline{\mathbf{X}}}\right)$$